Discretization of a Pseudo-Parabolic Equation

Abdelaziz TAAKILI, LMA, Univ. Pau, Bât IPRA, 64000 Pau

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We consider questions arising from approximate solution by discontinuous Galerkin finite element methods of the linear sobolev equation given by

$$u_t - \Delta u - \tau \Delta u_t = f \tag{1}$$

for $x \in \Omega$, $t \in J = (0,T]$, $\tau > 0$, where Ω is a bounded domain in $\mathbb{R}^d$, with Lipschitz boundary $\partial\Omega$. We consider homogenuous Dirichlet boundary condition for u, and the initiale condition :

$$u(x,0) = u_0(x), \quad x \in \Omega. \tag{2}$$

where $u_0 \in H_0^1(\Omega)$.

Equation of type (1)-(2) with one time derivative appearing in the highest order term are called pseudo-parabolic or sobolev equations, and arise from the study of flows of fluids through fissured rock, the two-phase flow in porous media with dynamical capillary pressure, thermodynamics, shear in second order fluids, and consolidation of clay (see Ewing [1] for reference), and other applications. The nature of such problems is transient and, therefore, an appropriate time stepping scheme has to be applied in numerical simulations to obtain an approximative solution. A flexible and efficient time discretization method is the Discontinuous Galerkin Finite Element Method (DGFEM) which is based on variational formulations of initial value problems.

This type of equation was first studied by Benjamin, Bona, and Mahony [2] as an alternative to the korteweg-de vries equations for describing unidirectionel long dispersive waves. Theorical results on existence, uniqueness, regularity, and decay at infinite time of solution for (1) are studied by many author (see Benjamin [2] and Tran [3] for reference). Numerical approximation based on finite difference, finite element, and spectral methods has been considered in [4, 5, 1].

In this talk, we introduce the DGFEM for the problem (1)-(2)(see Thomée [6] for parabolic equation). We derive an optimal error estimate that are explicit in polynomial order r, time step k and the regularity properties of the exact solution (see Schwab [7] for elliptic problem). Our theoretical results are completed by a series of numerical tests.

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Abdelaziz TAAKILI – abdelaziz.taakili@etud.univ-pau.fr
Laboratoire de Mathématiques Appliqués, Université de Pau, Bât IPRA, BP 1155, 64000 Pau