Proceedings in flow modelling around a cod-end net

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Improving the selectivity of trawling

⇒ By numerical simulations of the cod-end net

Main advantage: low cost of numerical simulations vs experimental measurements (at sea or in a tank),
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- A net model: discrete models. High number of meshes ⇒ Globalization techniques
- A model for the fishes: catch model or balls model
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- A net model: discrete models. High number of meshes ⇒ Globalization techniques
- A model for the fishes: catch model or balls model
- A fluid model ⇒ but complex geometry of the net ...

Question

How could the net be taken into account in the fluid model?
Existing fluid models

- Hypothesis of a uniform flow: Landweber's hypothesis
- Model of an axisymmetric porous membrane: B. Vincent (ECN PhD, 1996)
- Ring model: D. Marichal (2005)

Our contribution
- A 3D turbulent fluid model and its mathematical analysis
- Development of an axisymmetric code with the free software Freefem++
- Participation in an experimental campaign to collect hydrodynamical data
- Test and validation of the code by comparison with the experimental results
Existing fluid models

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1. A 3D turbulent fluid model
   - Experimental context
   - Our model
   - Averaged Navier-Stokes/Brinkman equations
   - Coupled system of equations
   - Theoretical result

2. Test of the model in a simple case
   - Axisymmetric problem
   - Simulations with FreeFem++

3. Conclusion
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Motivations

Finding a model that could:

- Control the passage of the fluid through the net
- Be applied in the 3D case (i.e. without the hypothesis of axisymmetric flow)
- Be applied to the case of a moving net
The model built at Boulogne-sur-Mer by G. Germain and J.V. Facq

Parameters of the net:

- Side mesh: 30mm
- Number of meshes on the perimeter: 36
- Length per weight: 1200m/kg
- Twine diameter: 1.5mm

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Profiles considered for the measures
LDV profiles of the velocity component $u_z$
Three features and their advantages

- A porous membrane model for the net ⇒ No more complex geometry of twines and nodes
- A penalization method to take the net and fishes into account: Navier-Stokes/Brinkman model with eddy viscosity ⇒ Possibility of 3D computations by the means of a Fictitious Domain Method: no complex mesh
- A closure equation for the TKE. This a kind of Reynolds Averaged Navier-Stokes model ⇒ To close the system
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- A closure equation for the TKE. This a kind of Reynolds Averaged Navier-Stokes model ⇒ To close the system
Averaged incompressible Navier-Stokes/Brinkman equations

- **Unknowns**: \((u - P)\) (mean velocity - modified pressure), \(k\) turbulent kinetic energy (TKE)

- Averaged incompressible Navier-Stokes/Brinkman equations with eddy viscosity

\[
\begin{align*}
\frac{\partial u}{\partial t} + (u \nabla)u - \nabla \cdot \sigma_t(u, P, k) + \frac{\nu_0}{K(x)} u &= 0, \\
\nabla \cdot u &= 0,
\end{align*}
\]

Where:

\[
\sigma_t(u, P, k) = -P \text{Id} + (\nu_0 + \nu_t)(\nabla u + (\nabla u)^t),
\]

\(P = p + \frac{2}{3} k\), modified pressure

\(\nu_0\) = kinematic viscosity of water.
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\begin{align*}
\sigma_t(u, P, k) &= -P \text{Id} + (\nu_0 + \nu_t)(\nabla u + (\nabla u)^t), \\
\nu_t &= C_1 \ell(x) k^{\frac{1}{2}}, \text{ eddy viscosity coefficient} \\
\ell(x) &= \text{mixing length}.
\end{align*}
\]
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\]

Where: \(K(x)\) is the permeability parameter.

\[
K(x) = \begin{cases} 
1 & \text{si } x \in \Omega_w, \\
\frac{1}{\epsilon} & \rightarrow +\infty \\
\epsilon & \rightarrow 0 \\
K_f & \text{si } x \in G_f \cup G_c, \\
K_f & \text{si } x \in G_n,
\end{cases}
\]
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**Fictitious domain method**: the fluid equations hold in the entire domain (C. S. Peskin (1972), Angot et al. (1999), Khadra et al. (2000), Carbou and Fabrie (2003))
Averaged incompressible Navier-Stokes/Brinkman equations with eddy viscosity

\[
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \nabla) \mathbf{u} - \nabla \cdot \sigma_t(\mathbf{u}, P, k) + \frac{\nu_0}{K(x)} \mathbf{u} = 0,
\]

\[
\nabla \cdot \mathbf{u} = 0,
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\[
\sigma_t(\mathbf{u}, P, k) = -P \, I_d + (\nu_0 + \nu_t)(\nabla \mathbf{u} + (\nabla \mathbf{u})^t)
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\end{align*}
\]

A closure equation for the TKE

\[
\frac{\partial k}{\partial t} + (\mathbf{u} \nabla) k = \nabla \cdot (\tilde{\nu}_t \nabla k) + \frac{\nu_t}{2} |\nabla \mathbf{u} + (\nabla \mathbf{u})^t|^2 - C_3 \frac{k^{3/2}}{\ell(x)}
\]

with \( \tilde{\nu}_t = C_2 \ell(x) k^{1/2} \) and \( C_2 \) adimensionalized constant.
Averaged incompressible Navier-Stokes/Brinkman equations with eddy viscosity

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\nabla \cdot \mathbf{u} &= 0, \\
\sigma_t(\mathbf{u}, P, k) &= -P \mathbb{1} + (\nu_0 + \nu_t)(\nabla \mathbf{u} + (\nabla \mathbf{u})^t)
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\]

Coupling parameter: \( \nu_t = C_1 \ell(x) k^{1/2} \)
Initial and boundary conditions

\[
\begin{align*}
\forall x \in \Omega, \quad & u(0, x) = u_0(x) \\
\forall x \in \Omega, \quad & k(0, x) = k_0(x) \\
\Gamma_i & = \mathbb{R}^3, \quad k|_{\Gamma_i} = 0, \\
0 & |_{\Gamma_i} = 0, \quad k|_{\Gamma_i} = 0, \\
\sigma_t(u, p, k) \cdot n|_{\Gamma_o} & = -\frac{1}{2}(u \cdot n)^{-1}(u - u_I) + (u \cdot n) u_I, \\
\tilde{\nu}_t \frac{\partial k}{\partial n}|_{\Gamma_o} & = -(u \cdot n)^{-1} k.
\end{align*}
\]
Theoretical result: Theorem

Hypothesis
Assume:

1. $\nu_t \in C^1$ and bounded,
2. $\tilde{\nu}_t \in C^1$ and bounded,
3. $\ell \in L^\infty$ and bounded,
4. $K \in C^1$ and bounded,
5. $u_0 \in L^2(\Omega)$, $\nabla \cdot u_0 = 0$, $u_0 \cdot n|_{\Gamma_i} = u_i$, $u_0 \cdot n|_{\Gamma_f} = 0$,
6. $k_0 \in L^1(\Omega)$
Then the coupled problem admits a solution \((u, P, k)\) on any time interval \([0, T]\) in the sense of the distributions, where

\[
\begin{align*}
u &\in L^2([0, T], (H^1(\Omega))^2) \cap L^\infty([0, T], L^2(\Omega)), \\
P &\in L^2([0, T] \times \Omega), \\
k &\in L^{4/3}([0, T], W^{1,4/3}(\Omega)) \cap L^\infty([0, T], L^1(\Omega)).
\end{align*}
\]

Moreover, there exists \(F(u_I, u)(t)\) such that the following energy equality holds for any \(t \in [0, T]\),

\[
\begin{align*}
\frac{1}{2} \frac{d}{dt} \int_\Omega |u(t, x)|^2 dx &+ \int_\Omega \nu_t(k(t, x), x)|\varepsilon(u)(t, x)|^2 dx + \\
\frac{1}{2} \int_{\Gamma_o} (u(t, x) \cdot n)^+ |u(t, x) - u_I|^2 d\sigma(x) &+ \\
\int_\Omega \frac{\nu_0}{K(x)} u(t, x) \cdot u(t, x) dx = F(u_I, u)(t).
\end{align*}
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Axisymmetric problem

- Hypothesis of an axisymmetric flow
- Cylindrical coordinates

\[
\begin{align*}
  x &= r \cos \theta, \\
  y &= r \sin \theta, \\
  z &= z.
\end{align*}
\]

with \( \{(r, z, \theta), r \in [r_{min}, r_{max}], z \in [z_{min}, z_{max}], \theta \in [0, \pi]\} \).
Decomposition of the net domain $G_n$ in 3 parts

$\Rightarrow$ To take into account the difference in permeability.
Numerical methods

- Finite elements method
- Numerical schemes:
  - Implicit scheme for the averaged Navier-Stokes/Brinkman equation
  - Semi-implicit scheme for the turbulent kinetic energy
- Iterative algorithm

**Algorithm**

1. Initialization of \((u, P)\) by solving a Stokes problem and \(k\) to a constant in the entire domain

2. For \(m=1, \text{Itmax}\)
   - Solving of the turbulent kinetic energy problem,
   - Solving of the Navier-Stokes/Brinkman problem.

End For
Mesh

Example of an unstructured body fitted mesh (10978 vertices - 21862 triangles)
Choice of the parameters

- $K_{\Omega_w} = 10000,$
- $K_{G_f} = 0.000001,$
- $K_{G_c} = 0.000001,$
- $K_{G_1^n} = 1,$
- $K_{G_2^n} = 5,$
- $K_{G_3^n} = 6,$
- Mesh : 10978 nodes ; 21862 triangles,
- Time step : 0.66667 s,
- $\ell(x)$ defined locally on each triangle as its higher side length,
- $C_1 = 0.1 ; C_2 = 0.05 ; C_3 = 0.03,$
- Thickness of the net : given by the minima of $u_z$ given by the LDV profiles.
Experimental vs numerical $u_z$ profiles at it 50
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Streamlines
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Level curves of $u_z$
Level curves of $u_z$
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Level curves of $k$
A stationary state is reached

- Residual computed for $u$
A stationary state is reached

- Residual computed for $k$

![Residual computed for $k$ graph]
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- We have a model that
  - Leads to satisfactory results in the axisymmetric case
    ⇒ Need some more experimental data, especially on the TKE
  - Can be generalized in 3D thanks to the Fictitious Domain Method
  - Make it easier to treat the problem of a moving net

- Current work
  - Implementation of the model in 3D,
  - Finding laws for the physical parameters in the model (depending on the mesh opening, the mesh side, ...)

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Any questions?