Shape and Topology Optimization by the Level Set Method. Application to Design of Compliant Mechanisms

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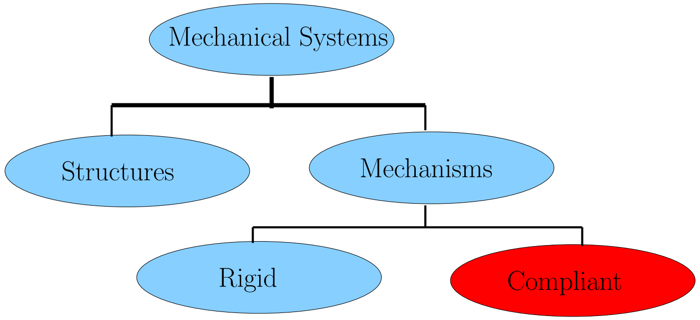
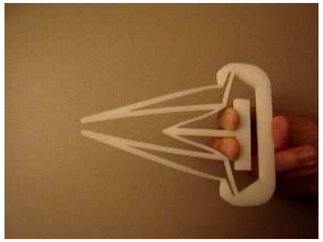
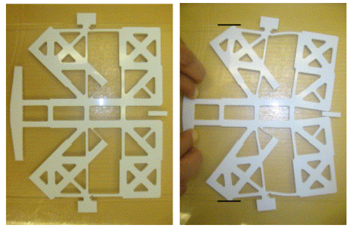
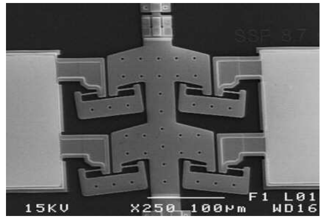
⊗ Classification



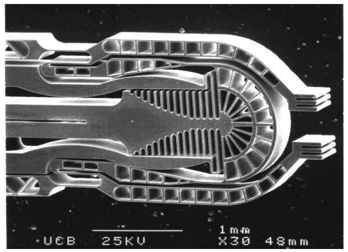


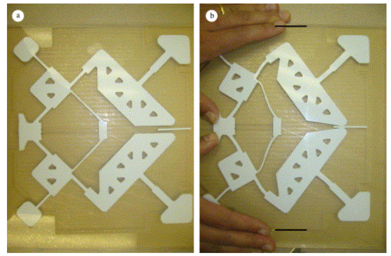




⊗ Definition

A flexure, or compliant mechanism, can be defined as a structure utilizing elastic deformation in some or all the parts of it to generate a specified mobility in the predetermined portion.





⊗ Application

- Micro domain: Micro Electro Mechanical System (MEMS)
- Macro domain: Medical, Automotive, Precision instrumentation, Manufacturing, Aerospace (Adaptative/Smart structures).

Topology synthesis approaches of compliant mechanisms

⊗ Kinematics synthesis approach (Her 1986, Howell and Midha 1994, Frecker and Kikuchi 1996).

⊗ Continuum synthesis approach (Ananthasuresh 1994).

- ⊗ Homogenization method (Bendsøe and Kikuchi 1988, Sigmund, Frecker, Nishiwaki, Allaire and Jouve,...).
- ⊗ Truss method (Sigmund 1997).
- ⊗ Flexible building blocks method (Bernardoni *et al.* 2004).
- ⊗ Level-set method:
  - G-equation: Markstein 1964
  - Osher & Sethian 1988
  - Sethian & Wiegmann 2000
  - Santosa 2000
  - Sethian & Santosa
  - Allaire, Jouve and Toader 2002.
  - Wang *et al.* 2003.

Level-set method for optimal synthesis of compliant mechanisms

⊗ Setting of the problem

Shape $\Omega \subset \mathbb{R}^d$ ($d = 2$ or 3) with boundary

$$\partial\Omega = \Gamma = \Gamma_{\mathrm{N}} \cup \Gamma_{\mathrm{D}},$$

with Dirichlet condition on $\Gamma_{\mathrm{D}} \neq \emptyset$, Neumann condition on Γ_{N}.

$$\begin{cases} -\mathrm{div}\,(Ae(u)) = f & \text{in } \Omega, \\ u = 0 & \text{on } \Gamma_D, \\ (Ae(u))n = g & \text{on } \Gamma_N, \end{cases}$$

with $e(u) = \frac{1}{2}(\nabla u + \nabla^T u)$, and A an homogeneous isotropic elasticity tensor.

⊗ The shape optimization problem

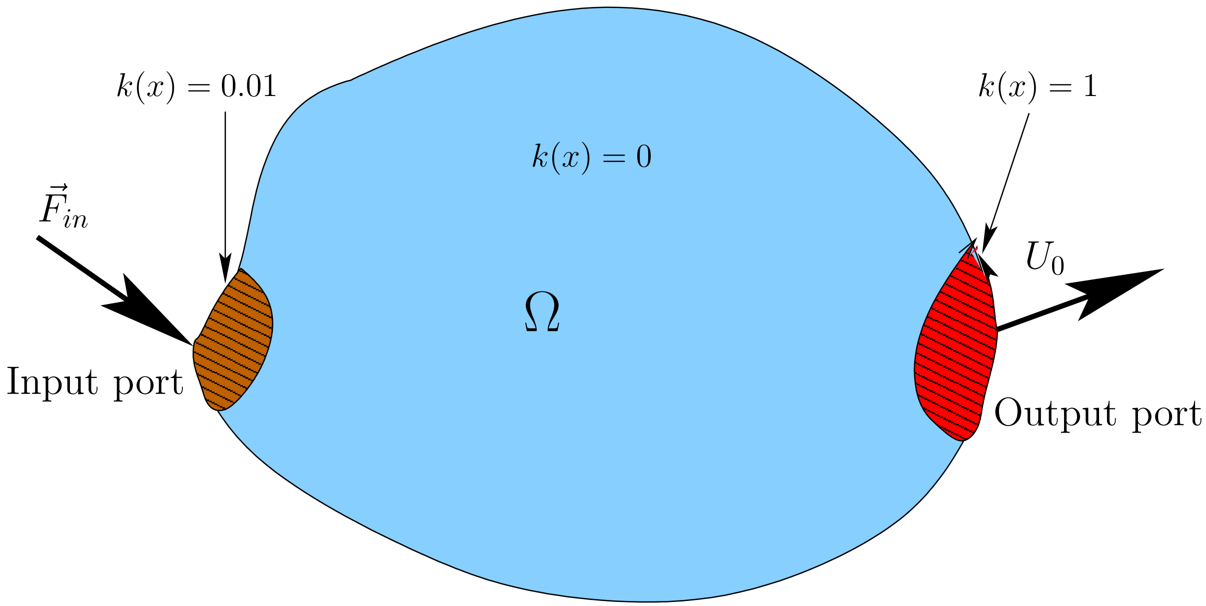
$$\inf_{\Omega \text{ admissible}} J(\Omega)$$

Objective Function

- A least square criteria for a targent displacement (SQ)

$$J_{SQ}(\Omega) = \left(\int_\Omega k(x)|u(x) - u_0(x)|^2 \, dx \right)^{\frac{1}{2}}$$

with a targent displacement u_0, and k a given weighting factor.



Design of compliant mechanisms

Objective Function

- Geometrical Advantage (GA) : $GA = \dfrac{\| \mathbf{u}_{out} \|}{\| \mathbf{u}_{in} \|} \left(= \dfrac{w_{out}.\mathbf{u}_{out}}{\| \mathbf{u}_{in} \|} \right)$

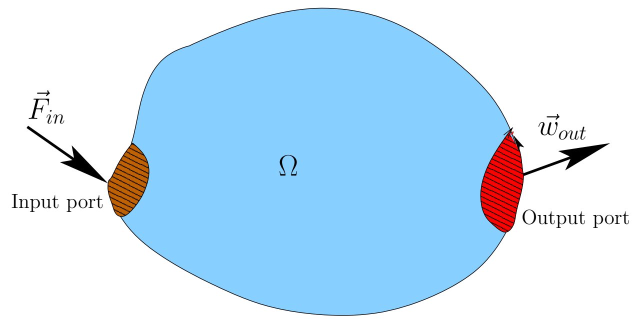
$$J_{GA}(\Omega) = -\frac{\displaystyle\int_\Omega \chi_{out}(x)(w_{out}(x), u(x))\ dx}{\left(\displaystyle\int_\Omega \chi_{in}(x)|u|^2(x)\ dx\right)^{1/2}}$$

χ_{out}, χ_{in}: the characteristic functions

w_{out} is added to indicate the direction of the preferred output displacement.

- Mechanical Advantage (MA) : $MA = \dfrac{\| \mathbf{F}_{out} \|}{\| \mathbf{F}_{in} \|} = \dfrac{w_{out}.\mathbf{F}_{out}}{\| \mathbf{F}_{in} \|} \left(\cong w_{out}.\mathbf{u}_{out} \right)$

$$J_{MA}(\Omega) = -\int_\Omega \chi_{out}(x)(w_{out}(x), u(x))\ dx$$



Principle of The Level-Set Method

⊗ Step 1: we compute shape derivatives (Framework of Murat-Simon) of the objective functions in a continuous framework.

Let Ω be a reference domain. Consider its variations

$\Omega_\theta = (\mathrm{Id} + \theta)\Omega$, with $\theta \in W^{1,\infty}(\mathbb{R}^d, \mathbb{R}^d)$.

Definition. The shape derivative of $J(\Omega_\theta)$ at Ω is the Fréchet differential of $\theta \to J((\mathrm{Id} + \theta)\Omega)$ at 0.

⊗ Step 2: Shape capturing method on a fixed mesh of a large box D. We model a shape by a level-set function ψ:

$$\begin{cases} \psi(x) = 0 & \Leftrightarrow \quad x \in \partial\Omega \cap D, \\ \psi(x) < 0 & \Leftrightarrow \quad x \in \Omega, \\ \psi(x) > 0 & \Leftrightarrow \quad x \in (D \backslash \bar{\Omega}). \end{cases}$$

The shape is varied by advecting the level-set function following the flow of the shape gradient (the transport equation is of Hamilton-Jacobi type):

$$\frac{\partial \psi}{\partial t} - j|\nabla_x \psi| = 0, \text{ with shape derivative } J^{'}(\Omega)(\theta) = \int_{\partial\Omega} j(u, p, n)\theta.n \, ds$$

Shape derivative of the least square criteria

$$J_{SQ}(\Omega) = \Big(\int_{\Omega} k(x)|u(x) - u_0(x)|^2 \, dx \Big)^{\frac{1}{2}}$$

$$\begin{aligned} J'_{SQ}(\Omega)(\theta) &= \int_{\partial\Omega} \Big(\frac{C_0}{2} k|u - u_0|^2 + Ae(p).e(u) \Big) \theta.n \, \mathrm{d}s \\ &- \int_{\Gamma_N} \Big(f.p + \frac{\partial(g.p)}{\partial n} + Hg.p \Big) \theta.n \, \mathrm{d}s, \end{aligned}$$

with the state u and the adjoint state p defined by

$$\begin{cases} -\mathrm{div}\ (Ae(p)) = -C_0 k(u - u_0) & \text{in } \Omega, \\ p = 0 & \text{on } \Gamma_D, \\ (Ae(p))n = 0 & \text{on } \Gamma_N, \end{cases}$$

and

$$C_0 = \Big(\int_{\Omega} k(x)|u(x) - u_0(x)|^2 \, dx \Big)^{-\frac{1}{2}}$$

Shape derivative of the Geometrical Advantage

$$J_{GA}(\Omega) = -\frac{\int_\Omega \chi_{out}(x)(w_{out}(x), u(x))\ dx}{\left(\int_\Omega \chi_{in}(x)|u|^2(x)\ dx\right)^{1/2}}$$

$$\begin{aligned} J'_{GA}(\Omega)(\theta) &= \int_{\partial\Omega} \Big(\Big(\frac{\chi_{out}(w_{out}, u)}{C_1} - \frac{\chi_{in}|u|^2}{2C_2}\Big) J_{GA}(\Omega) + Ae(p).e(u)\Big)\theta.n\ \mathrm{d}s \\ &- \int_{\Gamma_N} \Big(f.p + \frac{\partial(g.p)}{\partial n} + Hg.p\Big)\theta.n\ \mathrm{d}s, \end{aligned}$$

with the state u and the adjoint state p defined by

$$\begin{cases} -\mathrm{div}\ (Ae(p)) = \Big(\frac{\chi_{out} w_{out}}{C_1} - \frac{\chi_{in} u}{C_2}\Big) J_{GA}(\Omega) & \text{in } \Omega, \\ p = 0 & \text{on } \Gamma_D, \\ (Ae(p))n = 0 & \text{on } \Gamma_N, \end{cases}$$

$$C_1 = \int_\Omega (w_{out}(x), \chi_{out}(x) u(x))\ dx, \text{ and } C_2 = \int_\Omega \chi_{in}(x)|u|^2(x)\ dx$$

Shape derivative of the Mechanical Advantage

$$J_{MA}(\Omega) = -\int_{\Omega} \chi_{out}(x)(w_{out}(x), u(x))\; dx$$

$$\begin{aligned} J'_{MA}(\Omega)(\theta) &= -\int_{\partial\Omega} \Big(\chi_{out}(w_{out}, u) + Ae(p).e(u) \Big) \theta.n \;\mathrm{d}s \\ &- \int_{\Gamma_N} \Big(f.p + \frac{\partial(g.p)}{\partial n} + Hg.p \Big) \theta.n \;\mathrm{d}s, \end{aligned}$$

with the state u and the adjoint state p defined by

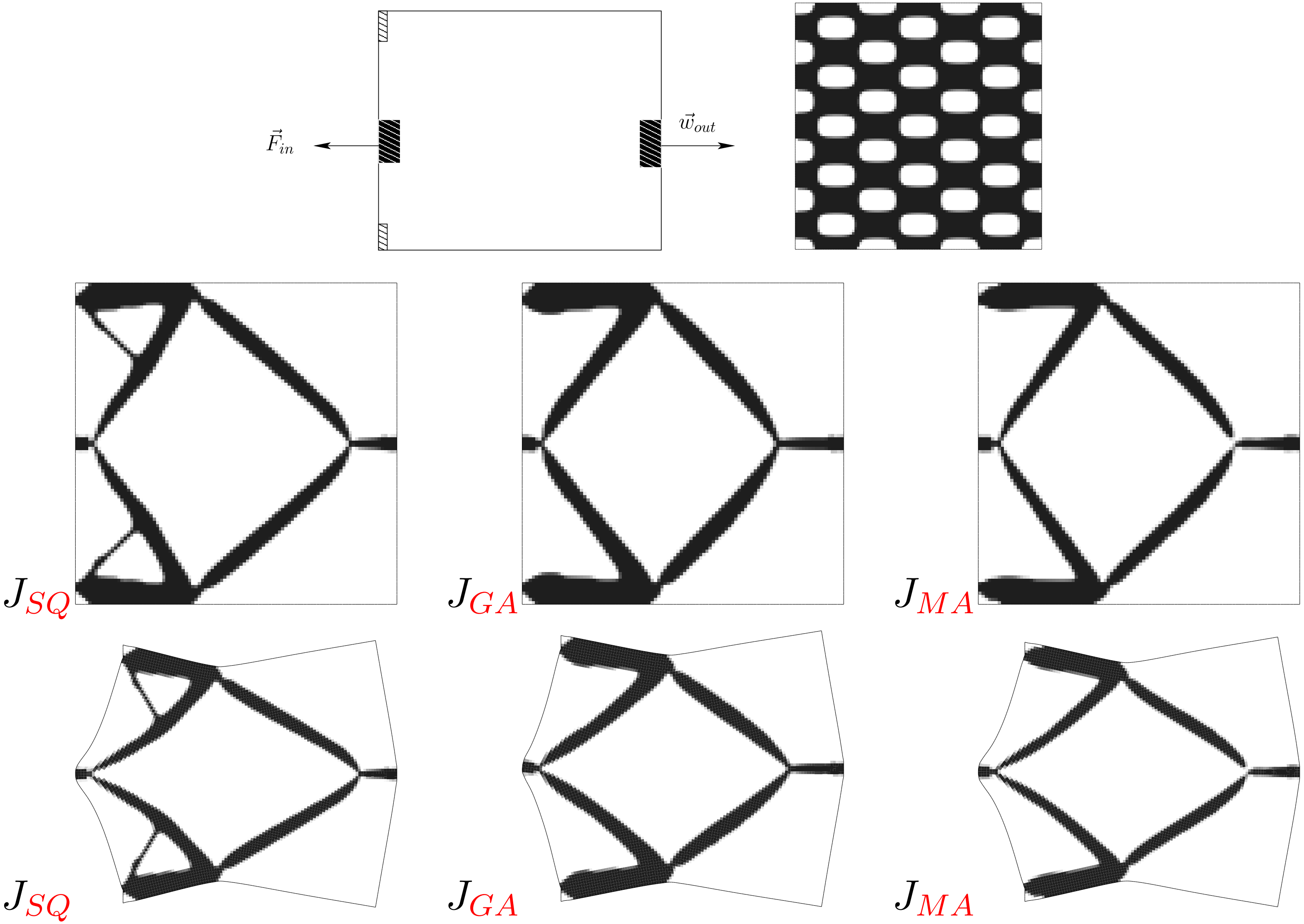
$$\begin{cases} -\mathrm{div}\ (Ae(p)) = -\chi_{out} w_{out} & \text{in } \Omega, \\ p = 0 & \text{on } \Gamma_D, \\ (Ae(p))n = 0 & \text{on } \Gamma_N, \end{cases}$$

Numerical Optimization Algorithm

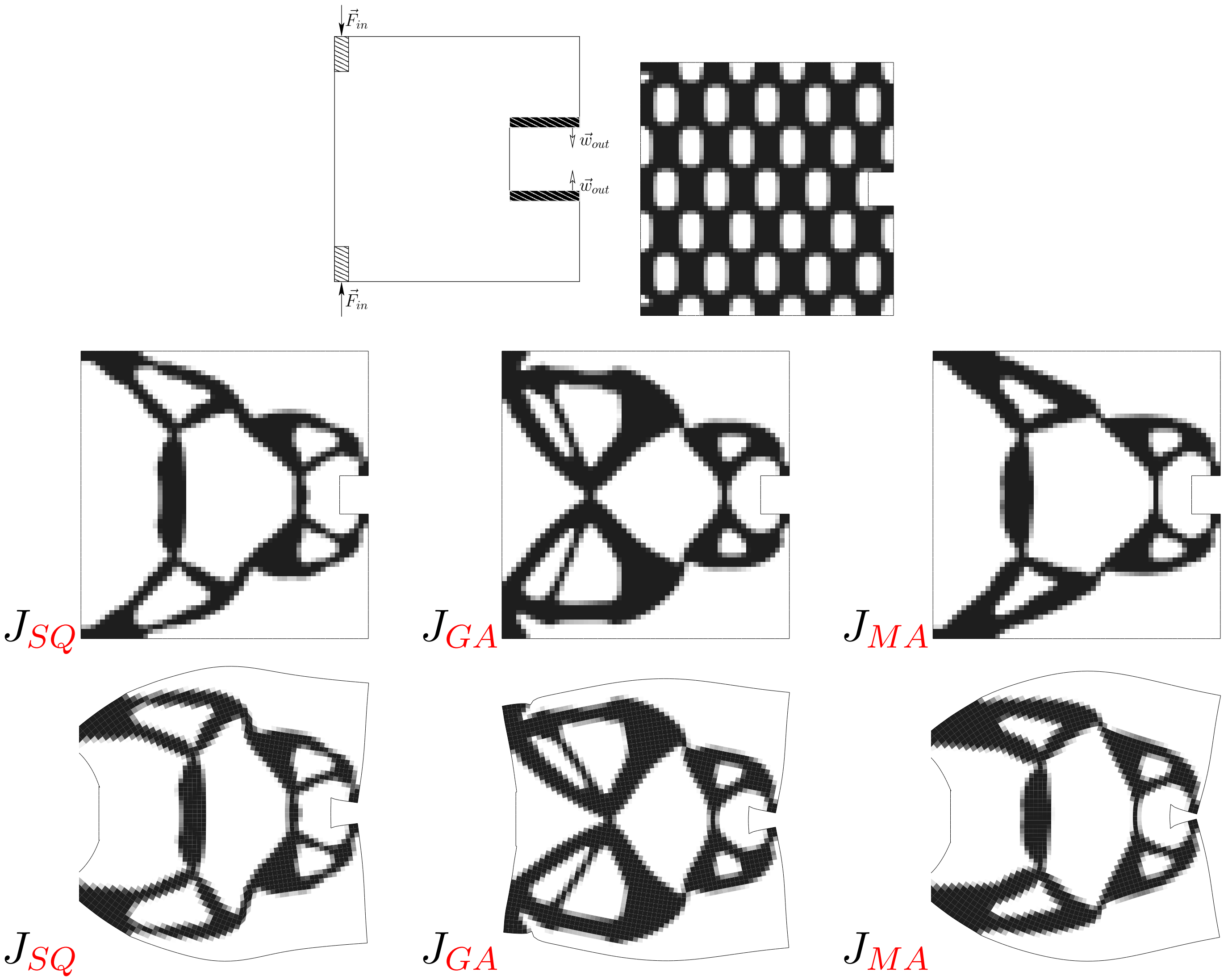
1. Initialization of the level set function ψ_0 (including holes).
2. Iteration until convergence for $k \geq 0$:
   (a) Computation of u_k and p_k by solving linear elasticity problem with the shape ψ_k.
   (b) Transport of the shape by V_k (Hamilton-Jacobi equation) to obtain a new shape ψ_{k+1}
3. Occasionally (stability reasons), re-initialization of the level set function ψ_{k+1} by solving

$$\begin{cases} \dfrac{\partial \psi_k}{\partial t} + \operatorname{sign}(\psi_{k-1})\big(|\nabla \psi_k| - 1\big) & = & 0 & \text{in } D \times \mathbb{R}^+, \\ \psi_k(t=0,x) & = & \psi_{k-1}(x) & \text{in } D. \end{cases}$$

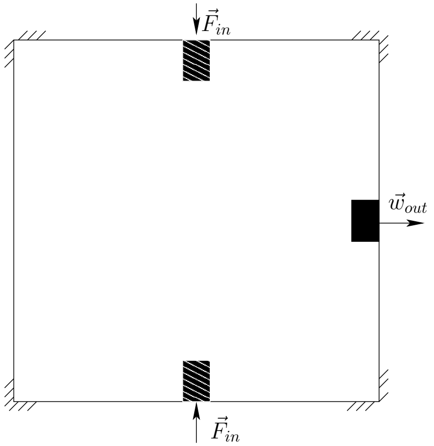
Example 1 A Displacement Inverter

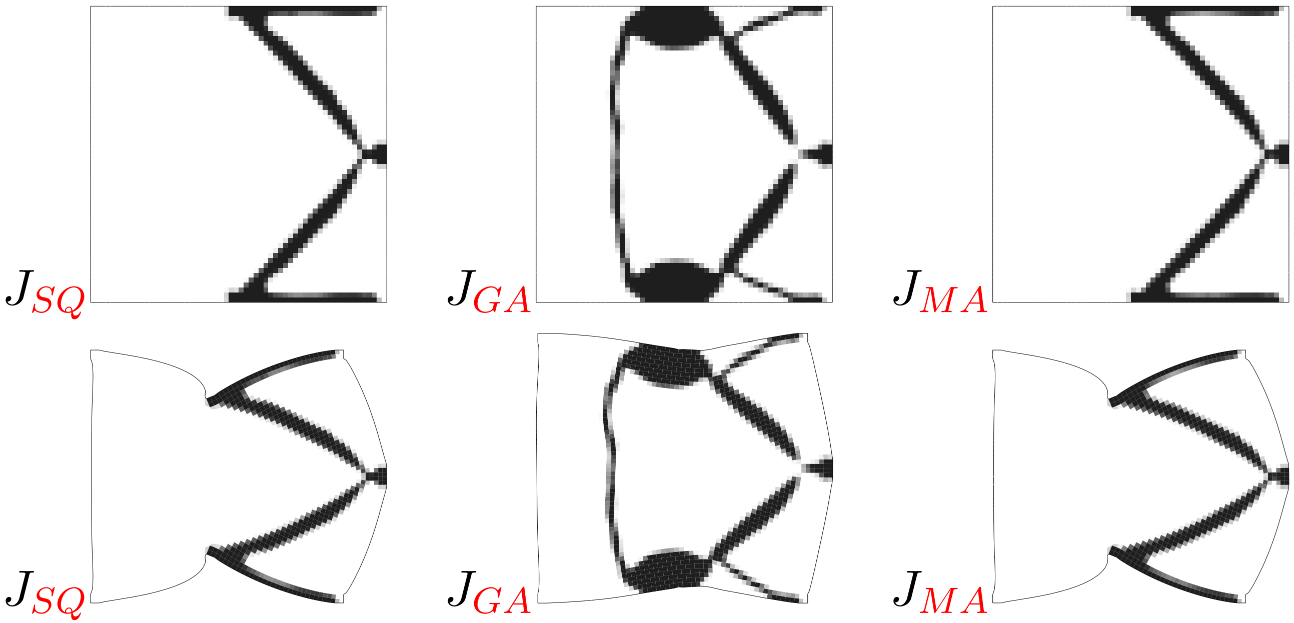


Example 2 Pull Micro-Gripper

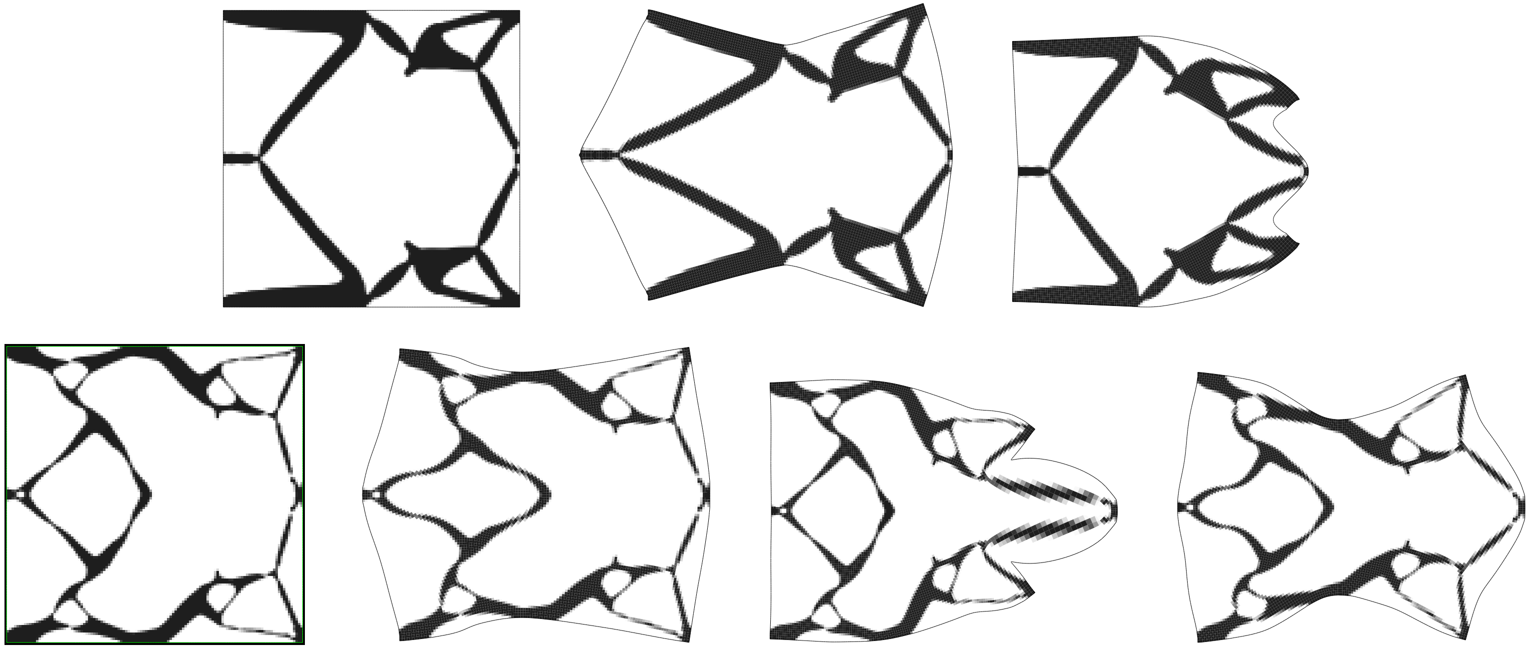


Example 3 Convection

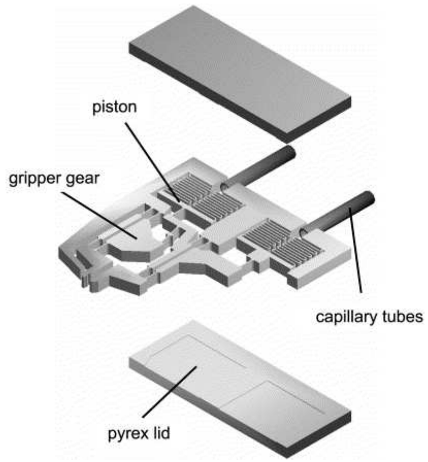


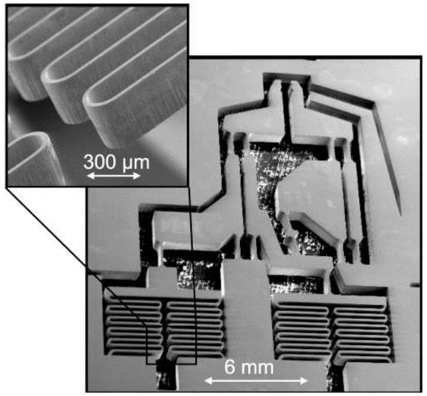


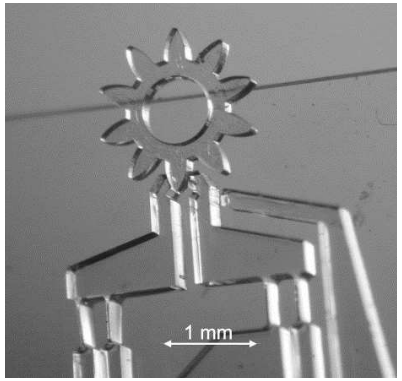
Multiple loads MEMS

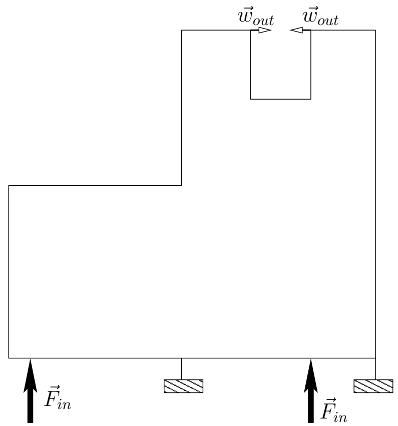


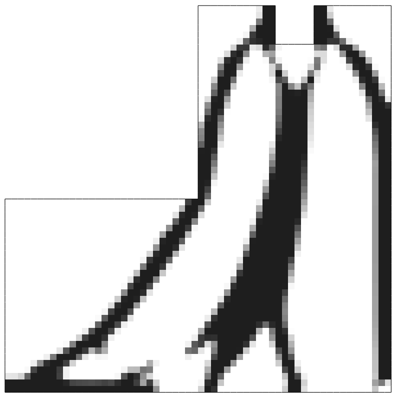
Novel micro-gripper for MEMS

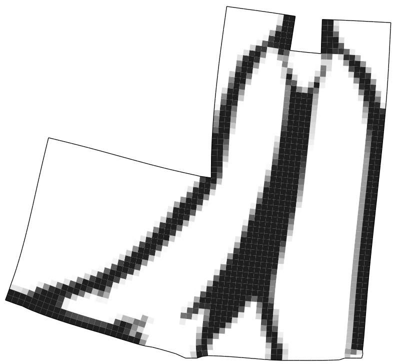


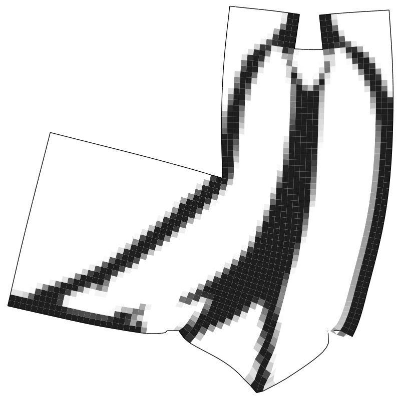












Butefisch, Seidemann and Buttgenbach., Sensors and Actuators A 97-98 (2002).

Conclusions and remaining issues

- Conclusions
  - Efficient method
  - Can handle non-linear model, design dependent loads and any smooth objective function
  - Many applications (Multi-physic problems, Electro-thermal compliant micromechanisms,...)
- Remaining issues
  - Influence of initialization (No nucleation of holes yet ⇒ topological gradient).
  - Insertion of specific constraints: $\mathbf{u}_{in} \leq \mathbf{u}_{in}^{max}$, $\mathbf{u}_{out} \leq \mathbf{u}_{out}^{max}$.
  - Weak material mimicking holes.