SCIENTIFIC MYSTERIES and SECRETS Resolved by Computation or Analytic vs Computational Math

Johan Hoffman and Claes Johnson

www.femcenter.org, www.bodysoulmath.org, www.fenics.org

NEW BOOK: www.bodysoulmath.org

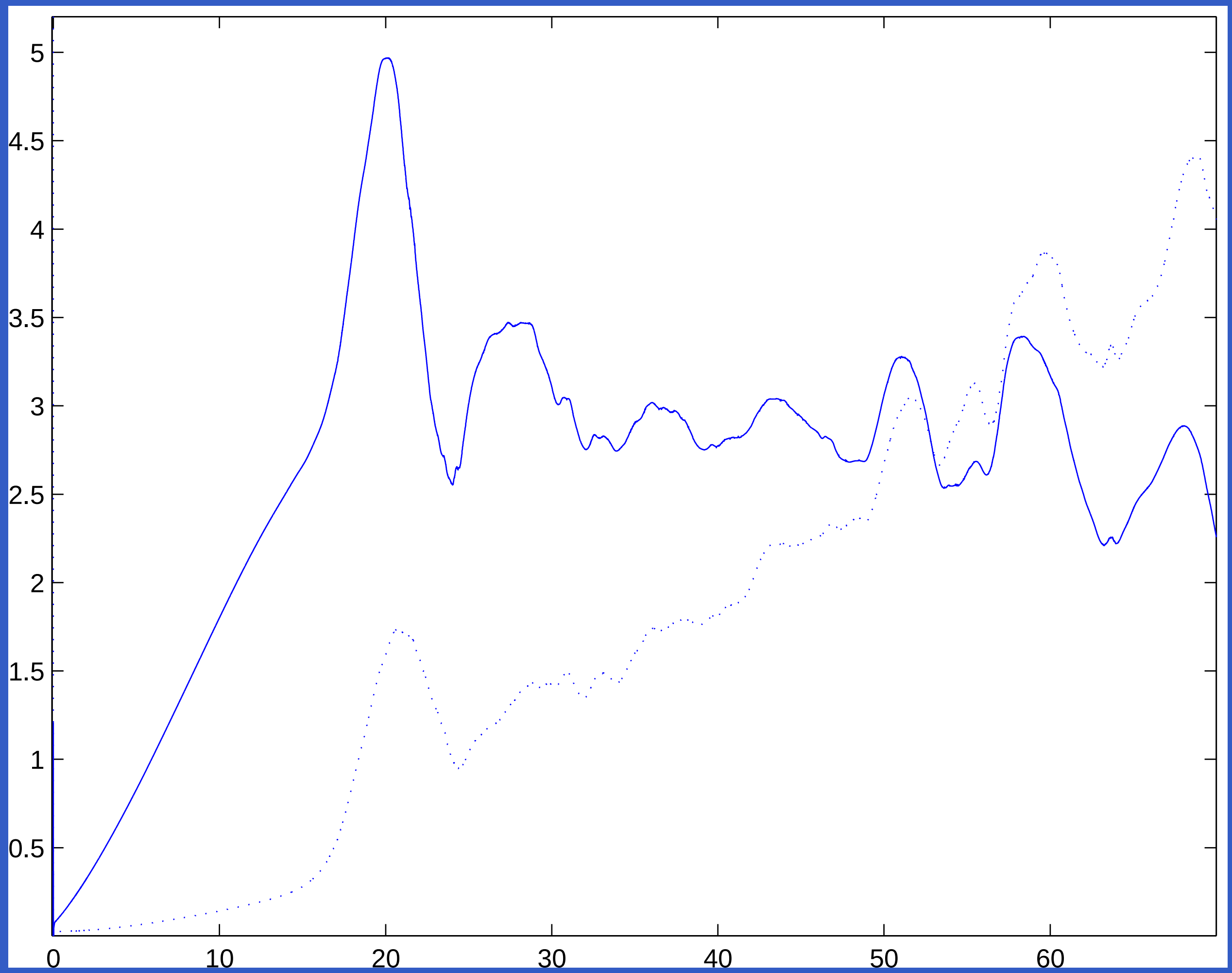
- Computational Turbulent Incompressible Flow
- Body$\&$Soul Vol IV
- Vol I-III Calculus Linear Algebra 2003
- Body $=$ Mathematical Computation
- Soul $=$ Mathematical Analysis
- Computational Thermodynamics Vol V 2007
- Computational Solid Mechanics Vol VI 2008
- Computational Quantum Mechanics Vol VII ...

SECRET of FLYING/SAILING

- LIFT vs DRAG

Pressure *Velocity* *Vorticity*

LIFT DRAG vs ANGLE of ATTACK



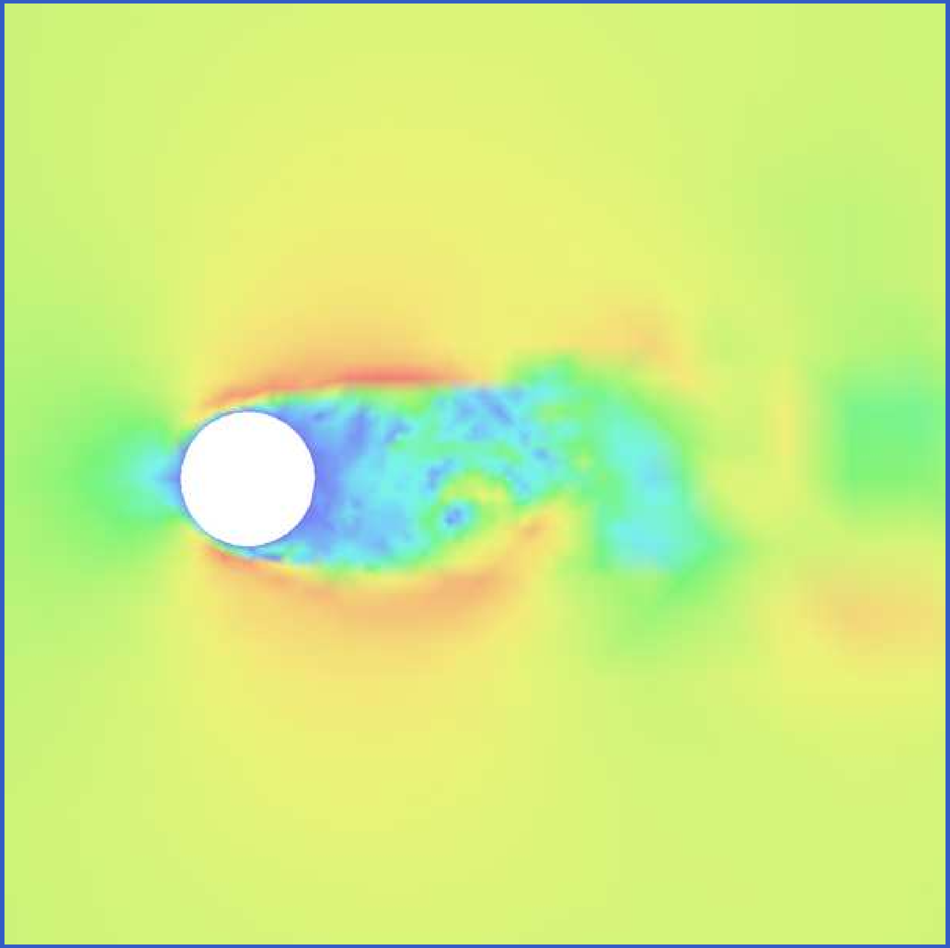
SECRETS of BALL SPORTS

- MAGNUS EFFECT: Rotating Ball
- DRAG CRISIS: Rough Surface-Friction

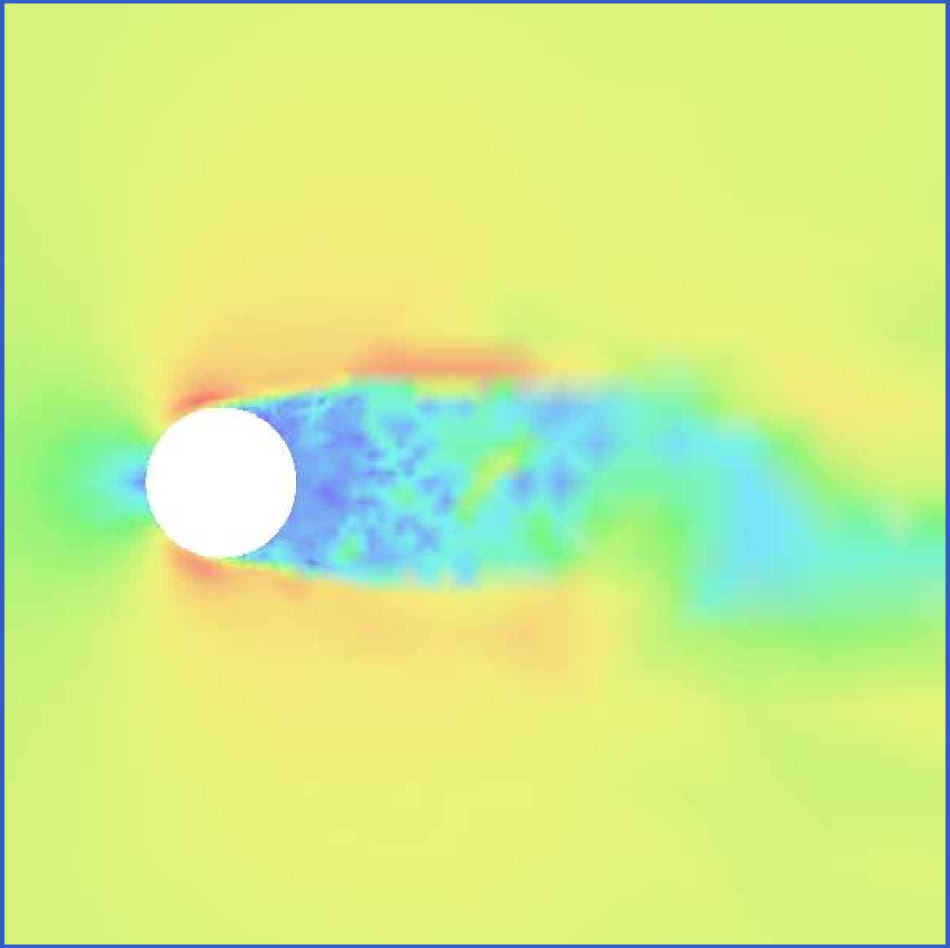
Carlos Free-Kick *Rotating Sphere*

Drag Crisis

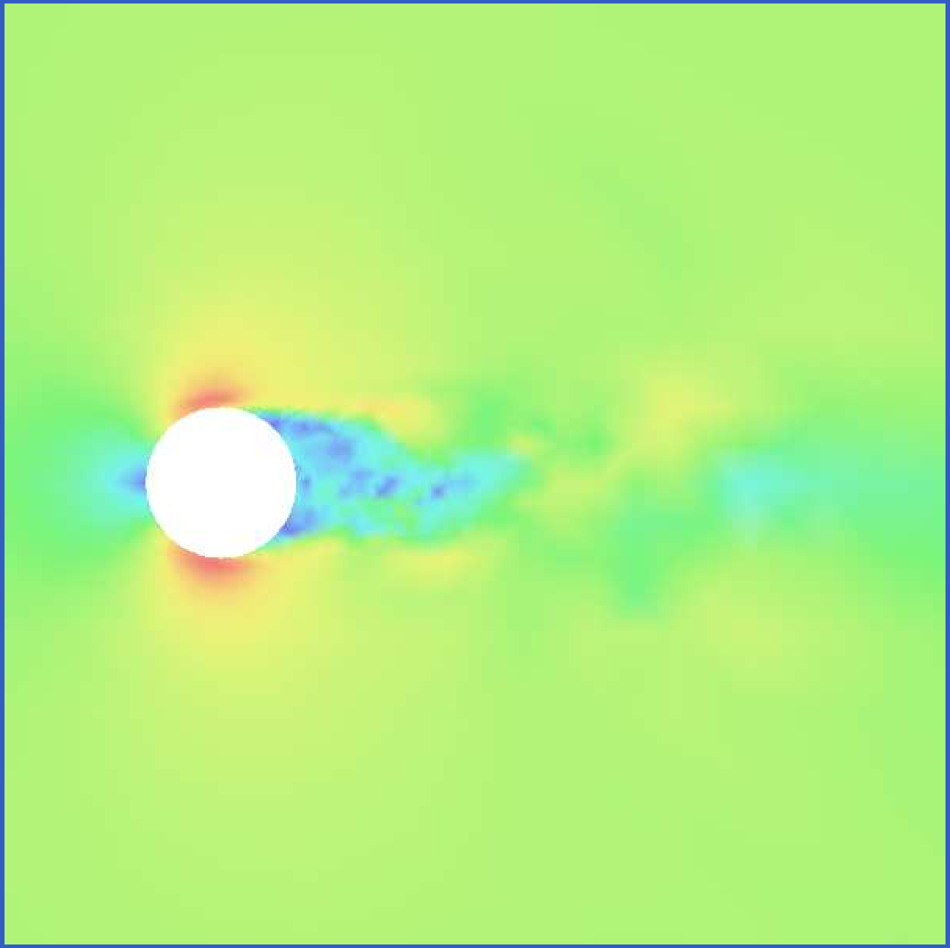
drag crisis; $\beta = 1$: $c_D \approx 1.0$



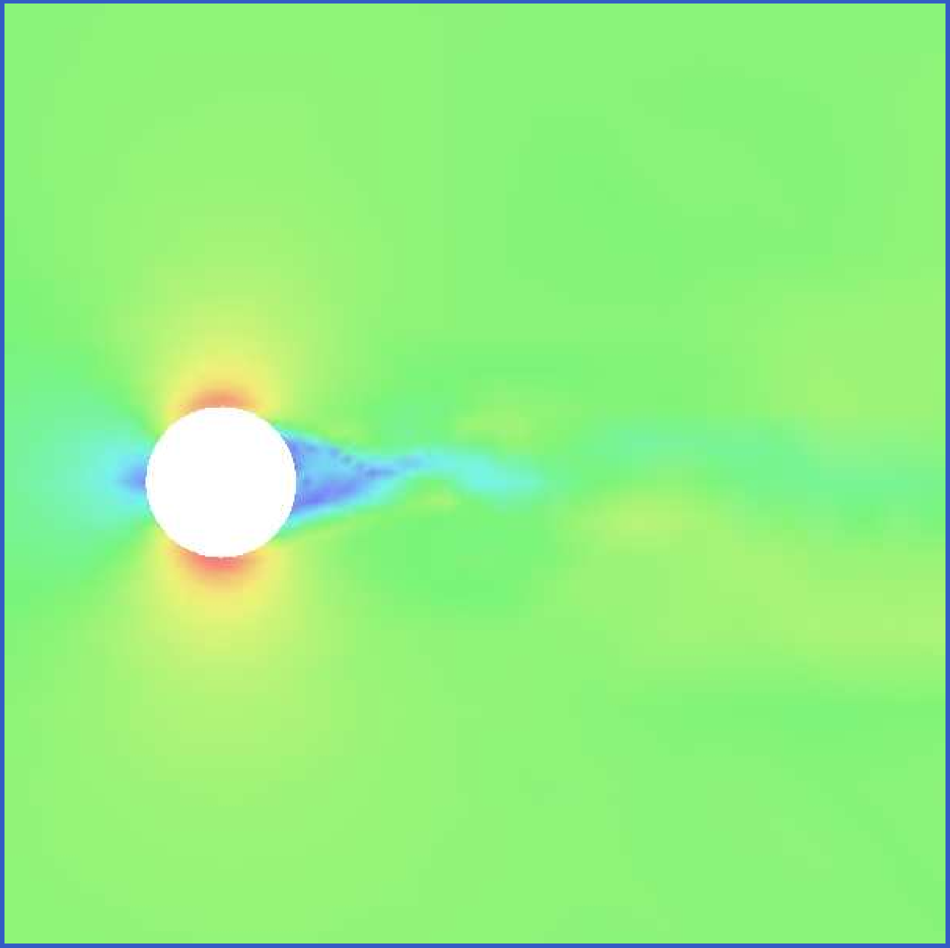
drag crisis; $\beta = 2 \times 10^{-2}$: $c_D \approx 0.7$



drag crisis: $\beta = 1 \times 10^{-2}$; $c_D \approx 0.5$



drag crisis; $\beta = 5 \times 10^{-3}$: $c_D \approx 0.45$



PDEs Without Exact Solutions

- PDE: EULER, Navier–Stokes,...
- Exact Solutions Non-Existent
- Computational Approx Weak Solutions Exists
- TURBULENT SOLUTIONS
- Output Error Control/Weak Uniqueness
- FINITE PRECISION-STABILITY
- $\$1$ Million CLAY PRIZE: Exist/Unique

EQUATIONS without SOLUTIONS!!

- TURBULENT SOLUTIONS
- APPROXIMATE WEAK SOLUTIONS
- EULER NAVIER-STOKES....

WHAT IS TURBULENCE?

- TURB $\equiv$ NON-EXIST EXACT SOL U

- $R(U) = 0$
- PW RESIDUAL SMALL : PW EXACT SOL

- $R(U) \neq 0$
- PW RESIDUAL LARGE: TURBULENCE

PYTHAGOREAN SOCIETY

- $x^2 = 2$ without EXACT (Rational) SOL
- Collapse of Pythagorean Society...
- EULER without EXACT SOL...
- TRYING vs SUCCEEDING (Olympics)
- cf LAWS of SOCIETY

COMPUTER CHANGES

- Mathematics: Methods-Research-Education?
- Mathematicians: NO!!
- Computational Mathematicians: YES!!
- Same for Physics, Chemistry, Technology??

ANALOG vs DIGITAL COMPUT.

- World $=$ Analog Computation
- Simulation $=$ Digital Computation
- FINITE PRECISION COMPUTATION

INFINITE vs FINITE PRECISION

- FORMALIST: Cantor: INFINITE PREC.
- CONSTRUCTIVIST: Brouwer: FINITE PREC.

- INFINITE PREC: Set of Real Numbers: $\mathbb{R}$
- FINITE PREC.: Computable Real Numbers

- INFINITE PREC: $x^2 = 2 \quad x \in \mathbb{R}$
- FINITE PREC.: $x^2 \approx 2 \quad x \in \mathbb{Q}$

Kurzweil: SINGULARITY 2045

- Kurzweil: Synthesizer....
- Moore's law:
- Computational power doubles every 18 months
- Digital Revolution
- Computational Technology Blow Up 2045
- Infinite Speed of Development

Computational Technology CT

- Technology *FOR* Computation: Mathematics
- Technology *WITH* Computation:
- Computational Solid/Fluid Mechanics...
- Electromagnetics...Quantum Mechanics
- Physics, Chemistry, Biology,...Cosmology
- Economy...Information...Medicin...
- COMPUTATIONAL CALCULUS CC

CT CC vs Mathematics Education

- Calculus: Basis of Trad. Technology
- CC: Basis of New Technology

- China: 400.000 Engineers/year $\times$ CC $= \infty$??
- Exponential Blow Up??

- Europe: Tradition: No Change
- Calculus: Education Stable for 300 years

MYSTERIES-PARADOXES

- D’ALEMBERT 1752
- LOSCHMIDT 1876
- SOMMERFELD 1900

- ANALYTICAL vs COMPUTATIONAL
- MATHEMATICS

D'ALEMBERT: INVISCID FLOW

- ANALYTICAL POTENTIAL SOLUTION:
- Zero Drag: NOT TRUE
- COMPUTATIONAL: Non-Zero Drag: TRUE
- COVER-UP: Very Small Viscosity (PRANDTL)

LOSCHMIDT: HAMILTONIAN SYST

- Analytical: Reversible: NOT TRUE
- Computational: Irreversible: TRUE
- COVER-UP: Statistical Mechanics (Boltzmann)

SOMMERFELD: COUETTE FLOW

- Analytical: Stable: NOT TRUE
- Computational: Unstable: TRUE

- COVER-UP: Couette Too Simple (Schlichting)

RESOLUTION: TURBULENCE

- Analytical Turbulence: IMPOSSIBLE
- Computational Turbulence: POSSIBLE
- Laminar Exact Solutions UNSTABLE
- Non-Existence of Exact Solutions

POTENTIAL SOLUTION $\Delta\phi = 0$

- ϕ VELOCITY POTENTIAL
- $U = \nabla\phi$ FLOW VELOCITY
- Irrotational Incompressible Inviscid Stationary
- ZERO-DRAG
- WHAT's WRONG??

EULER: U Velocity, P Pressure

$$\nabla \cdot U = 0 \qquad \text{Incompressible}$$

$$\dot{U} + U \cdot \nabla U + \nabla P = 0 \qquad \text{Momenentum}$$

$$U \cdot n = 0 \qquad \text{(Slip BC)}$$

$$U(\cdot, 0) = U^0 \qquad \text{(Initial)}$$

- Formally Reversible: $t \to -t$, $U \to -U$
- Parameterfree Model of the World: (Einstein)
  - $(1/c = G = h = \nu = 0)$
  - INVISCID $+$ INCOMPRESSIBLE

Conservation of Energy: EULER

Multiplication of Momentum by U:

$$\dot{U} + U \cdot \nabla U + \nabla P = 0 \quad U$$

Integration in space-time $\rightarrow$ Conserv. of Kinetic Energy:

$$\frac{1}{2}\|U(T)\|^2 = \frac{1}{2}\|U(0)\|^2.$$

- Allows Perpetuum Mobile
- WRONG!!
- WHY??

Conservation of Vorticity $\omega = \nabla \times U$

Taking vorticty of momentum eq:

$$\dot{\omega} + u \cdot \nabla \omega - (\omega \cdot \nabla) U = 0, \quad \nabla \cdot \omega = 0$$

- If $\omega(0) = 0$, then $\omega(T) = 0$
- Irrotational flow stays Irrotational
- WRONG!!
- WHY??

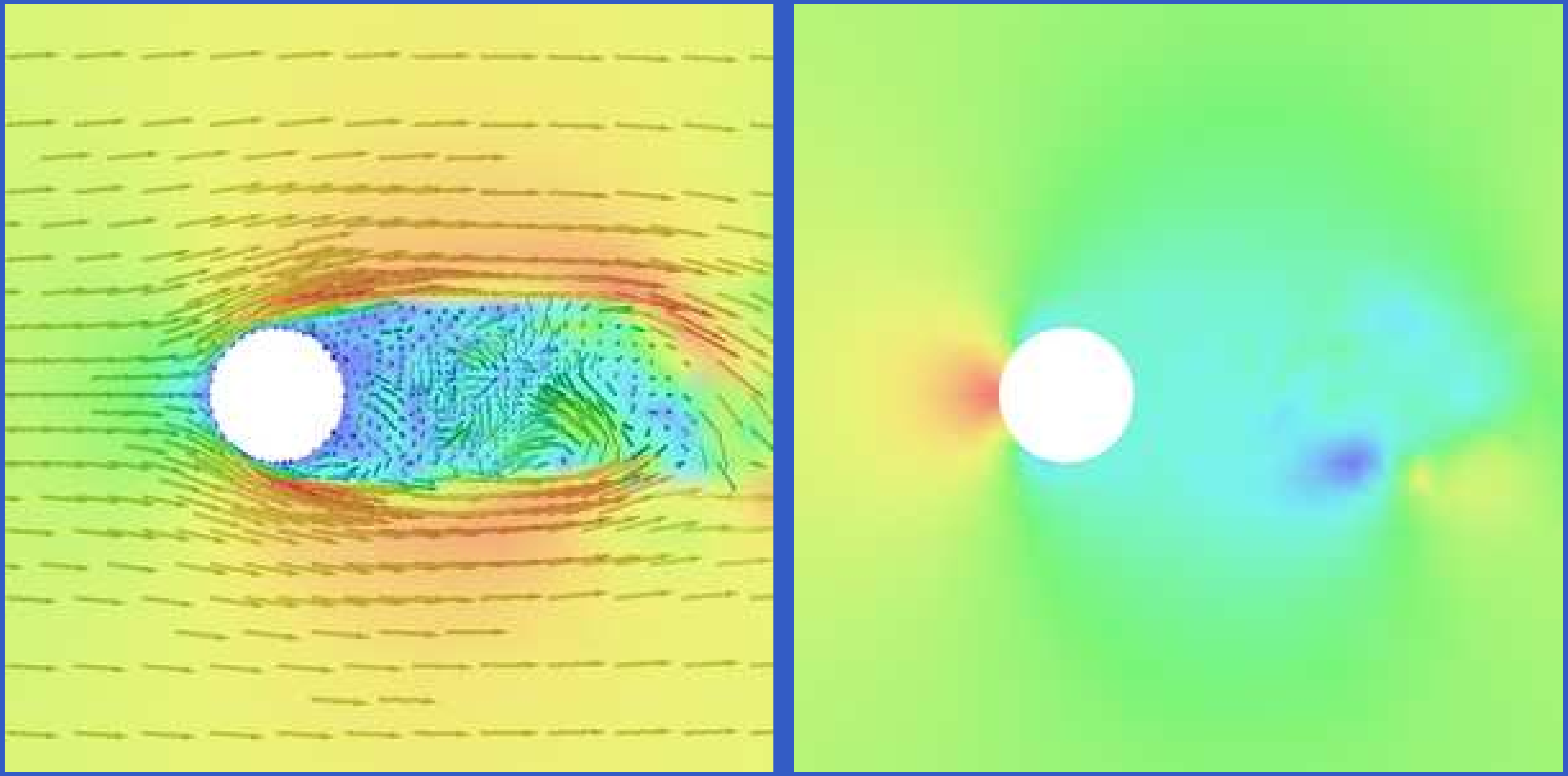
RESOLUTION: EULER/G2

- Incompressible EULER
- G2 General Galerkin: Stabilized FEM
- Navier-Stokes: SMALL VISCOSITY
- SLIP BC: SKIN FRICTION SMALL

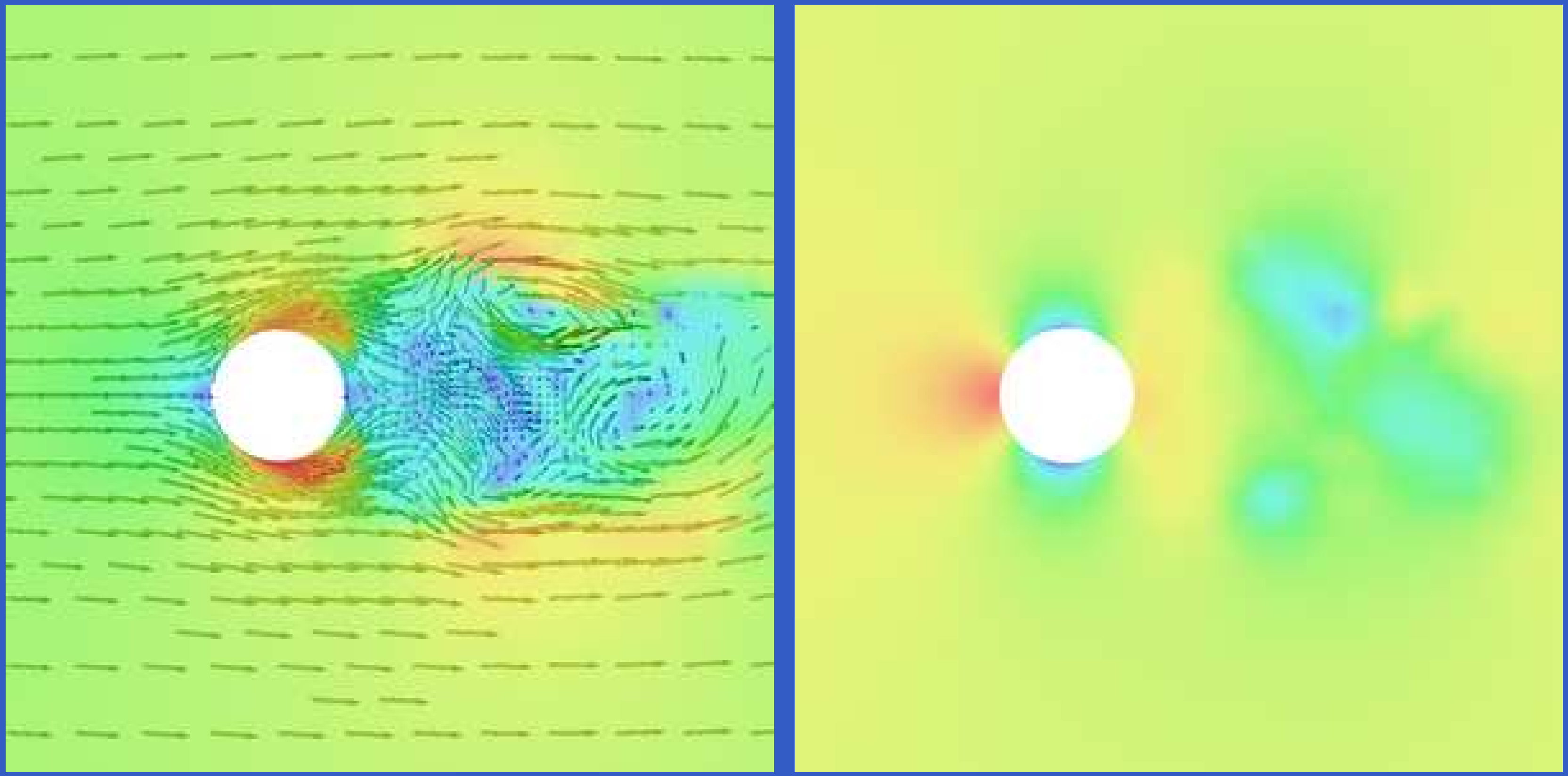
- 3 MYSTERIES in 1 Shot.

Guadalupe Aug 20 1999

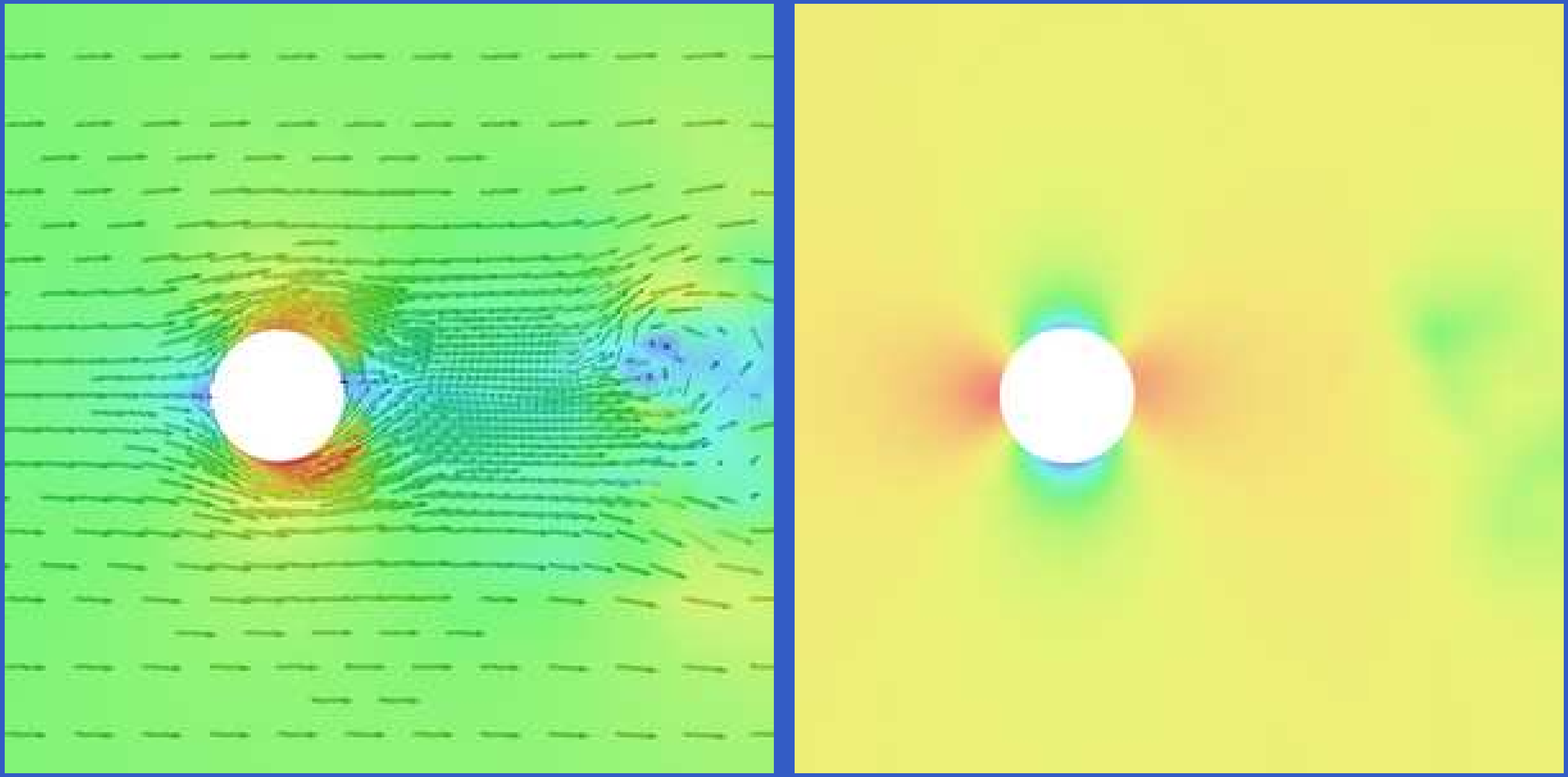
Velocity & pressure: t=0.0: $c_D = 1.03$



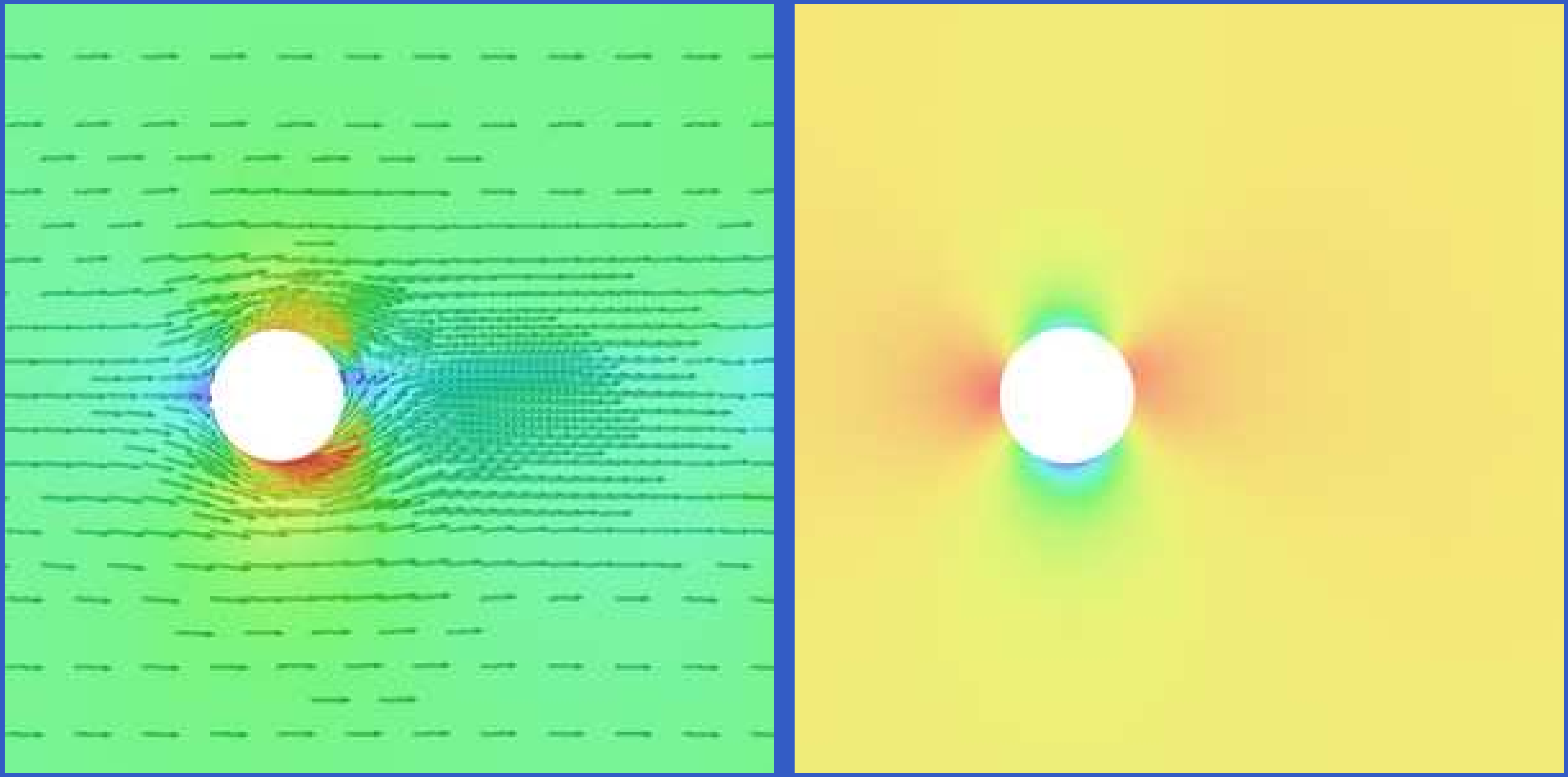
Velocity & pressure: t=0.25: $c_D = 0.06$



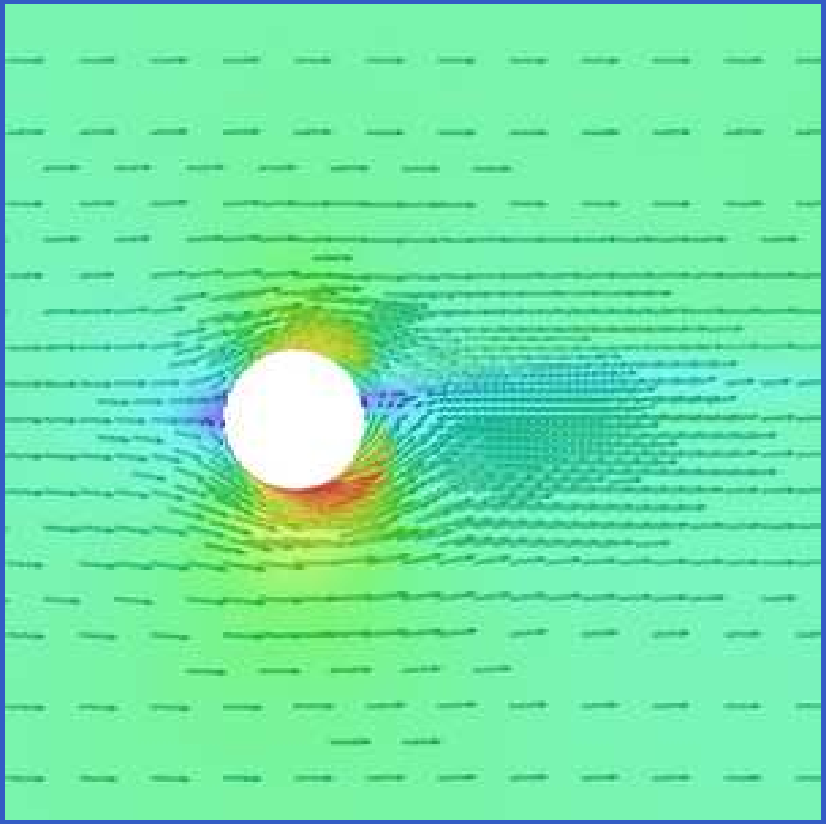
Velocity & pressure: t=0.5: $c_D = 0.10$

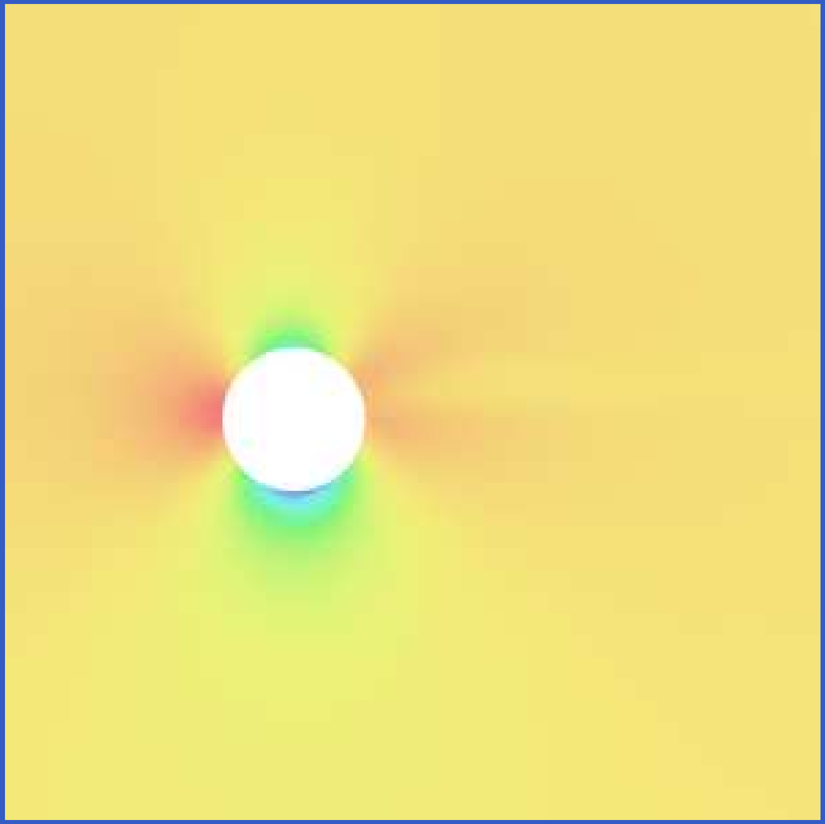


Velocity & pressure: t=0.75: $c_D = 0.15$

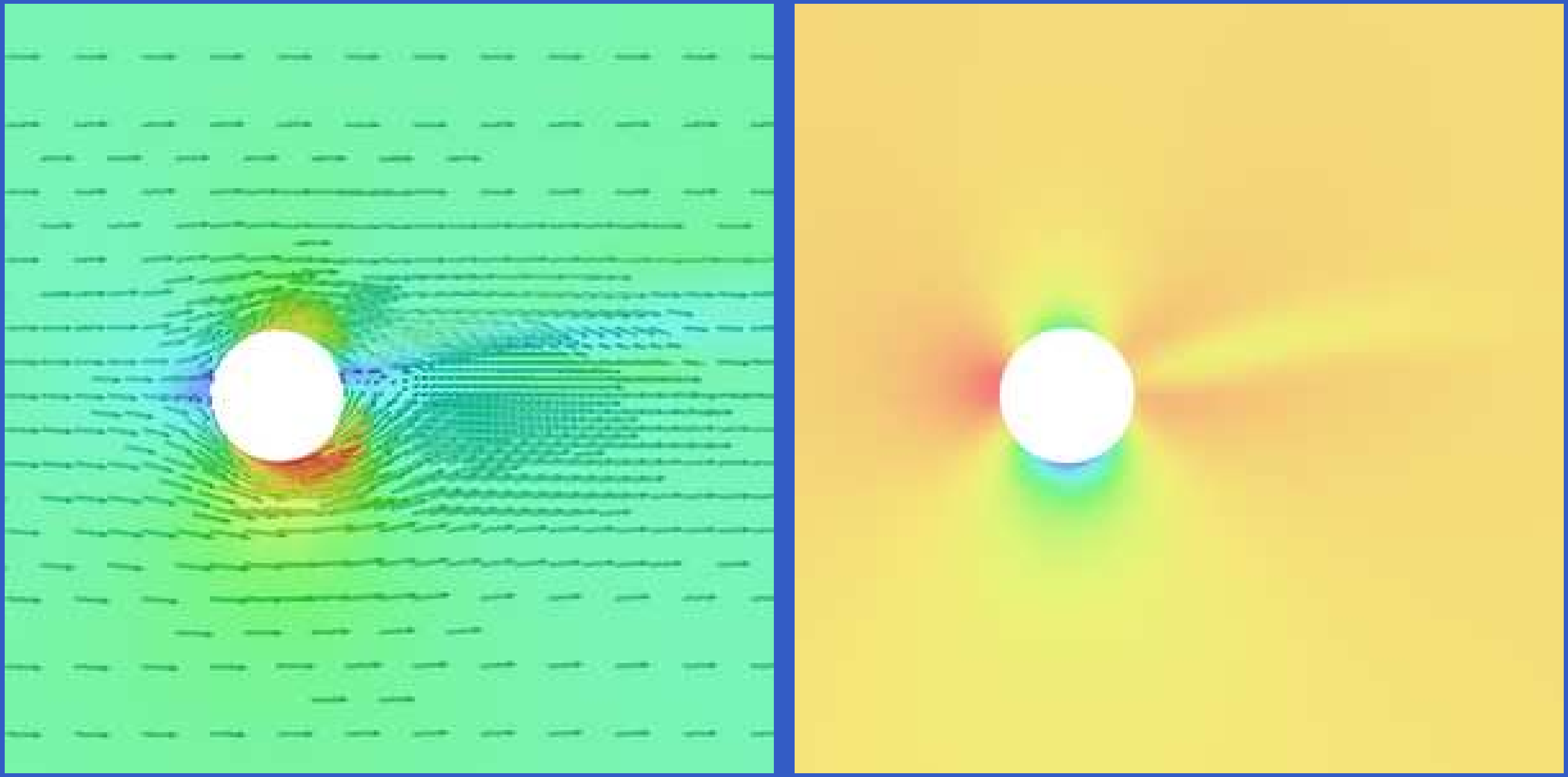


Velocity & pressure: t=1.0: $c_D = 0.22$

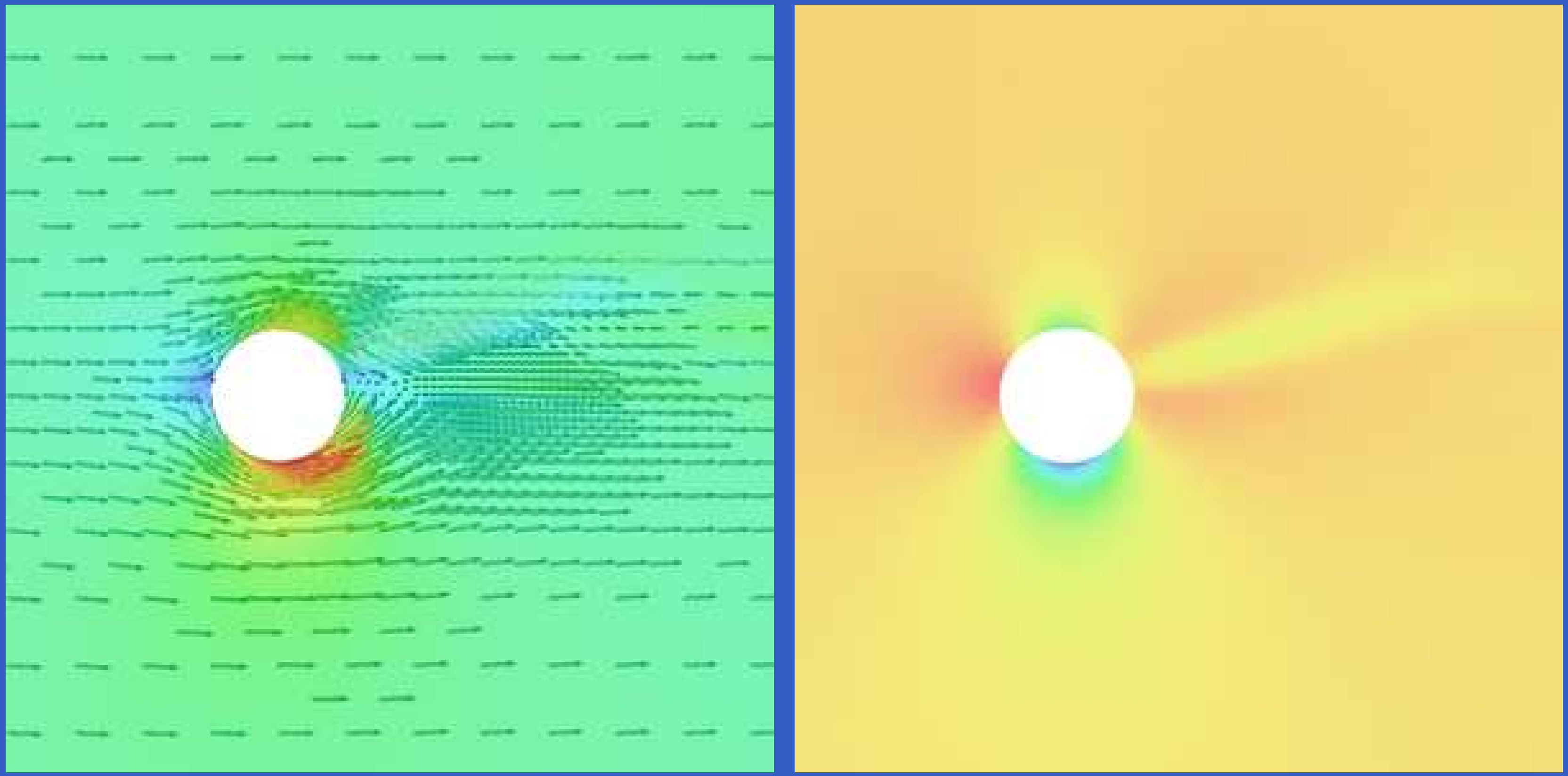




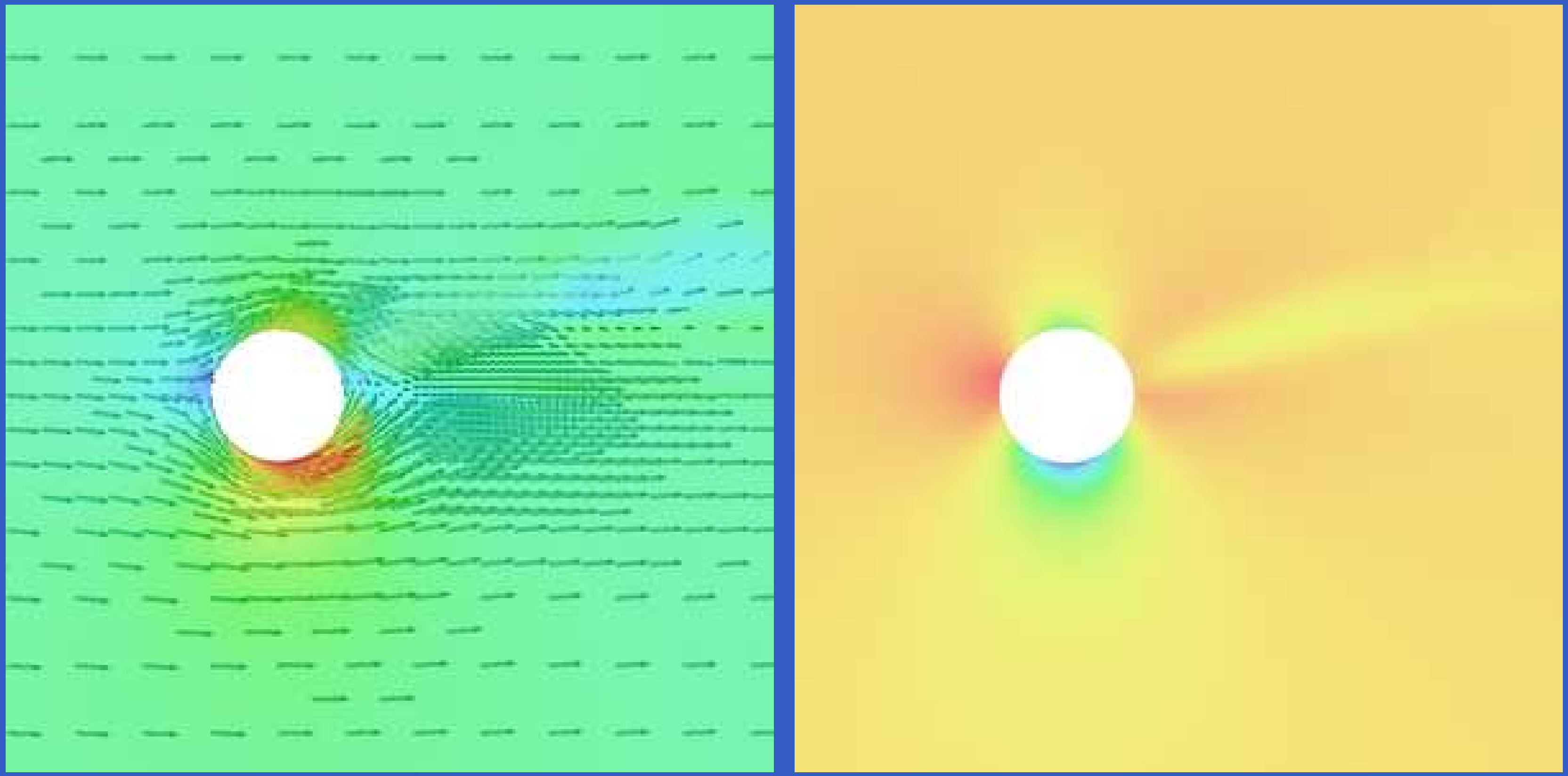
Velocity & pressure: t=1.25: $c_D = 0.25$



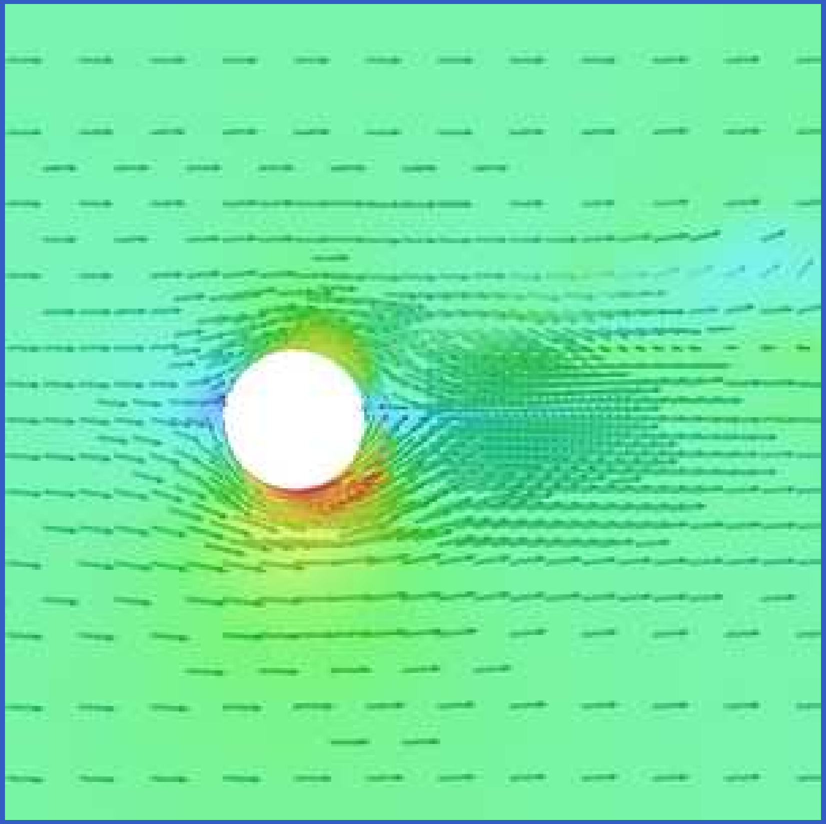
Velocity & pressure: t=1.5: $c_D = 0.28$

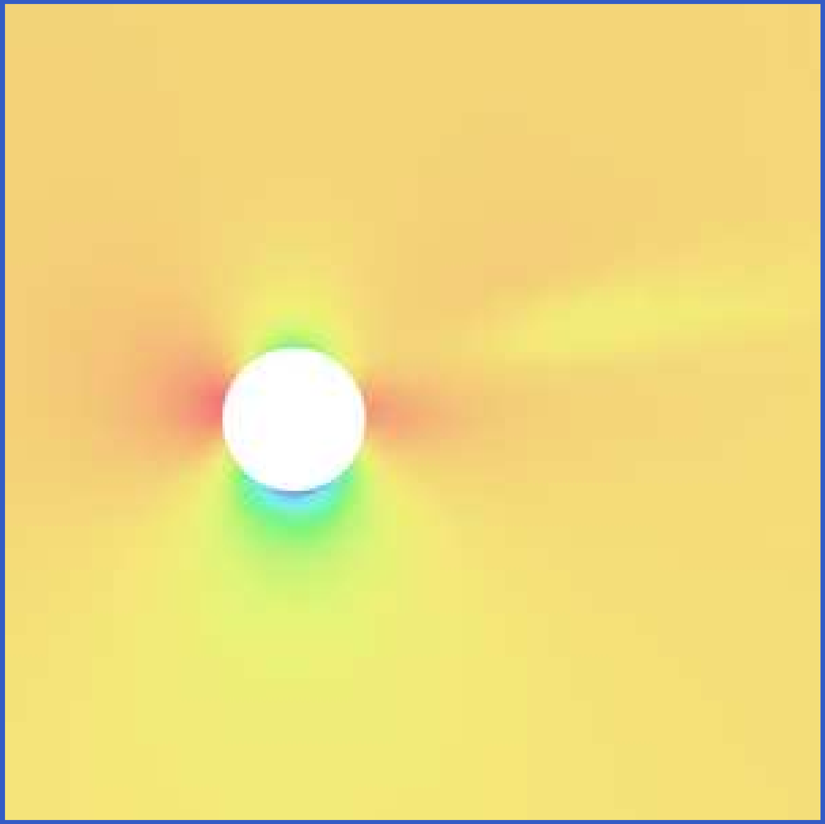


Velocity & pressure: t=1.75: $c_D = 0.36$

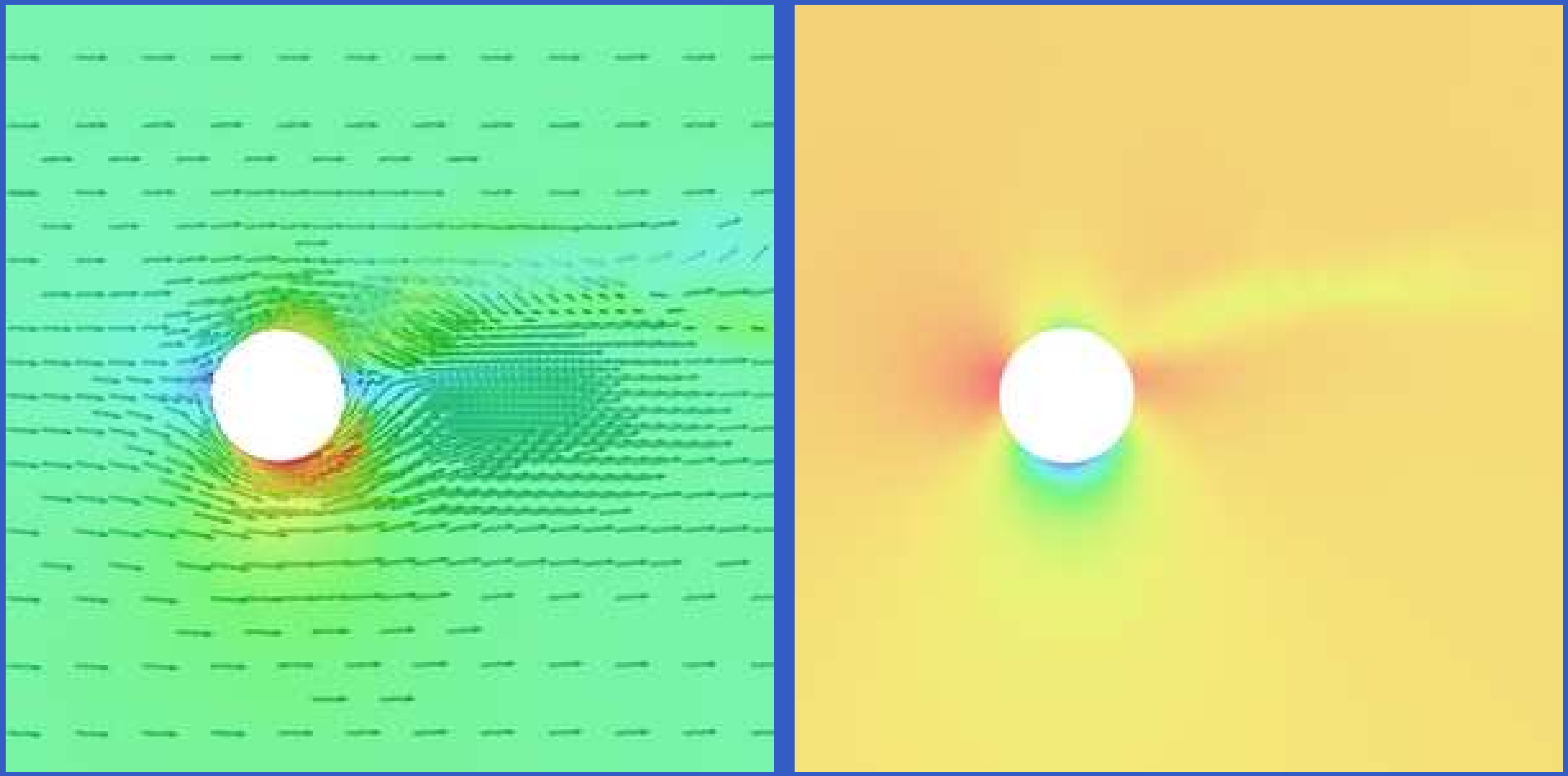


Velocity & pressure: t=2.0: $c_D = 0.51$

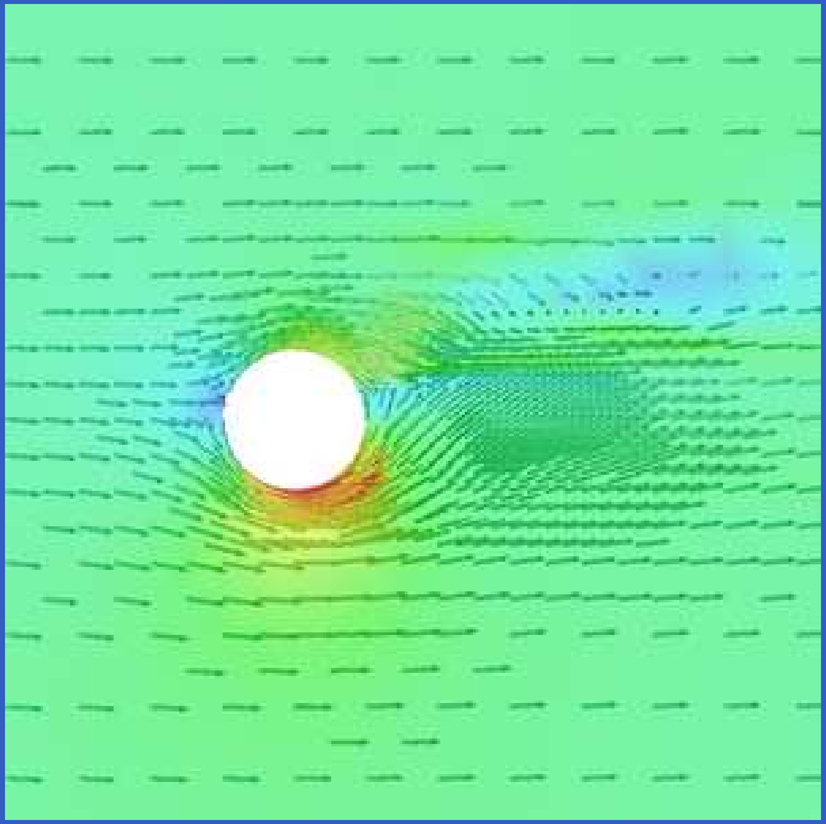


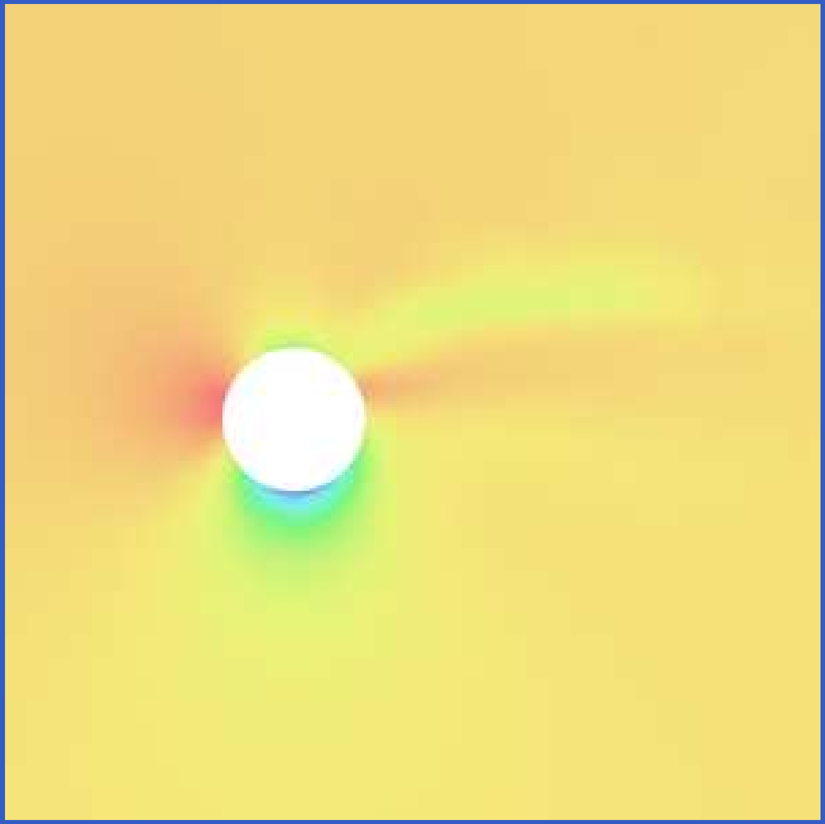


Velocity & pressure: t=2.25: $c_D = 0.78$

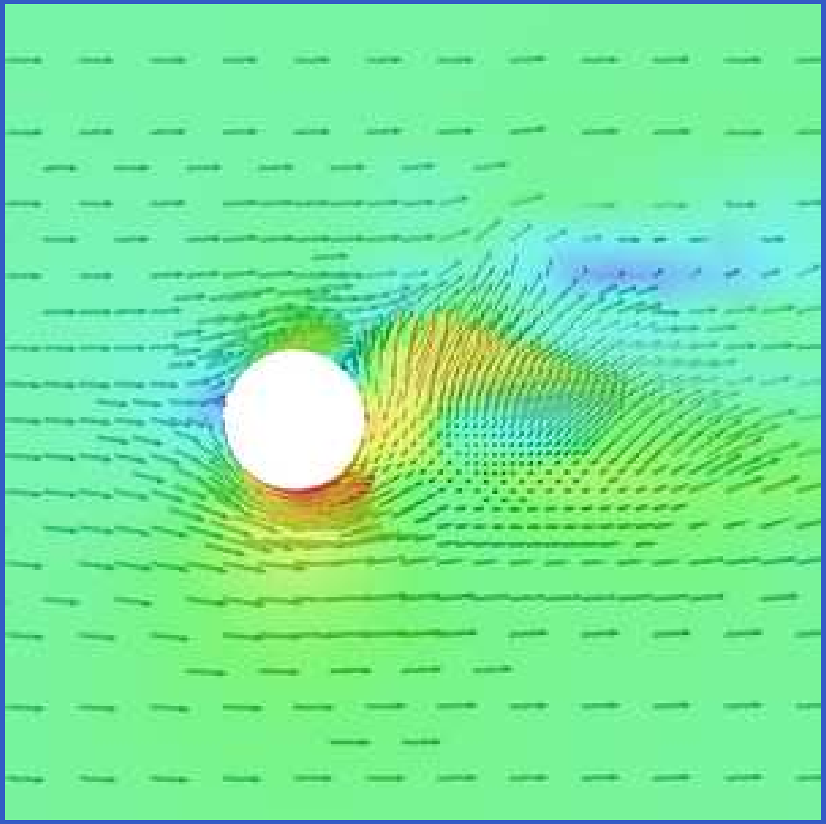
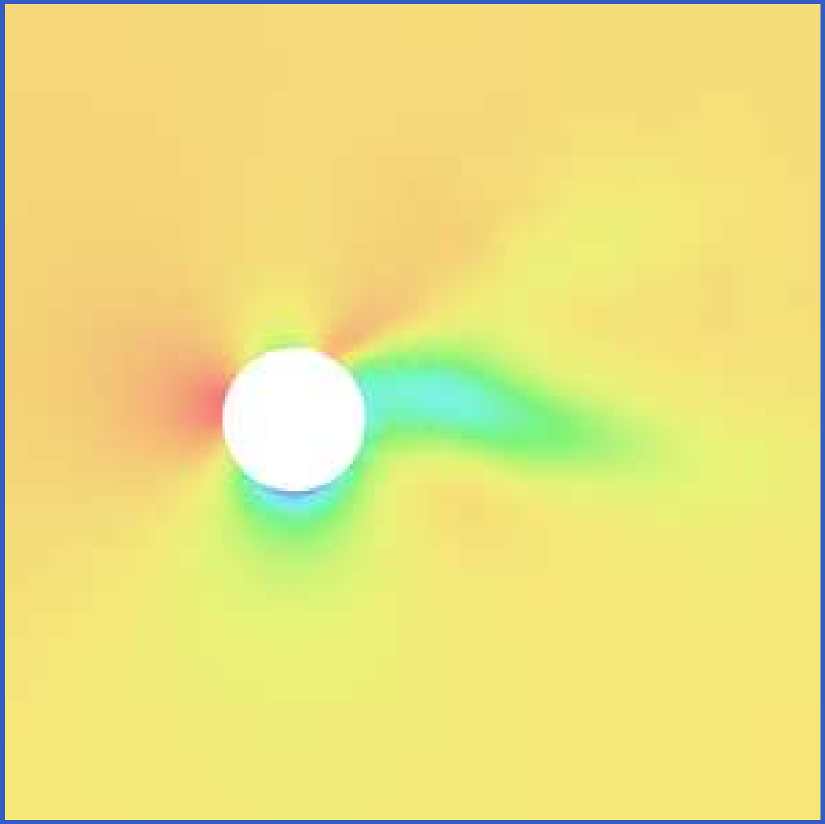


Velocity & pressure: t=2.5: $c_D = 1.14$

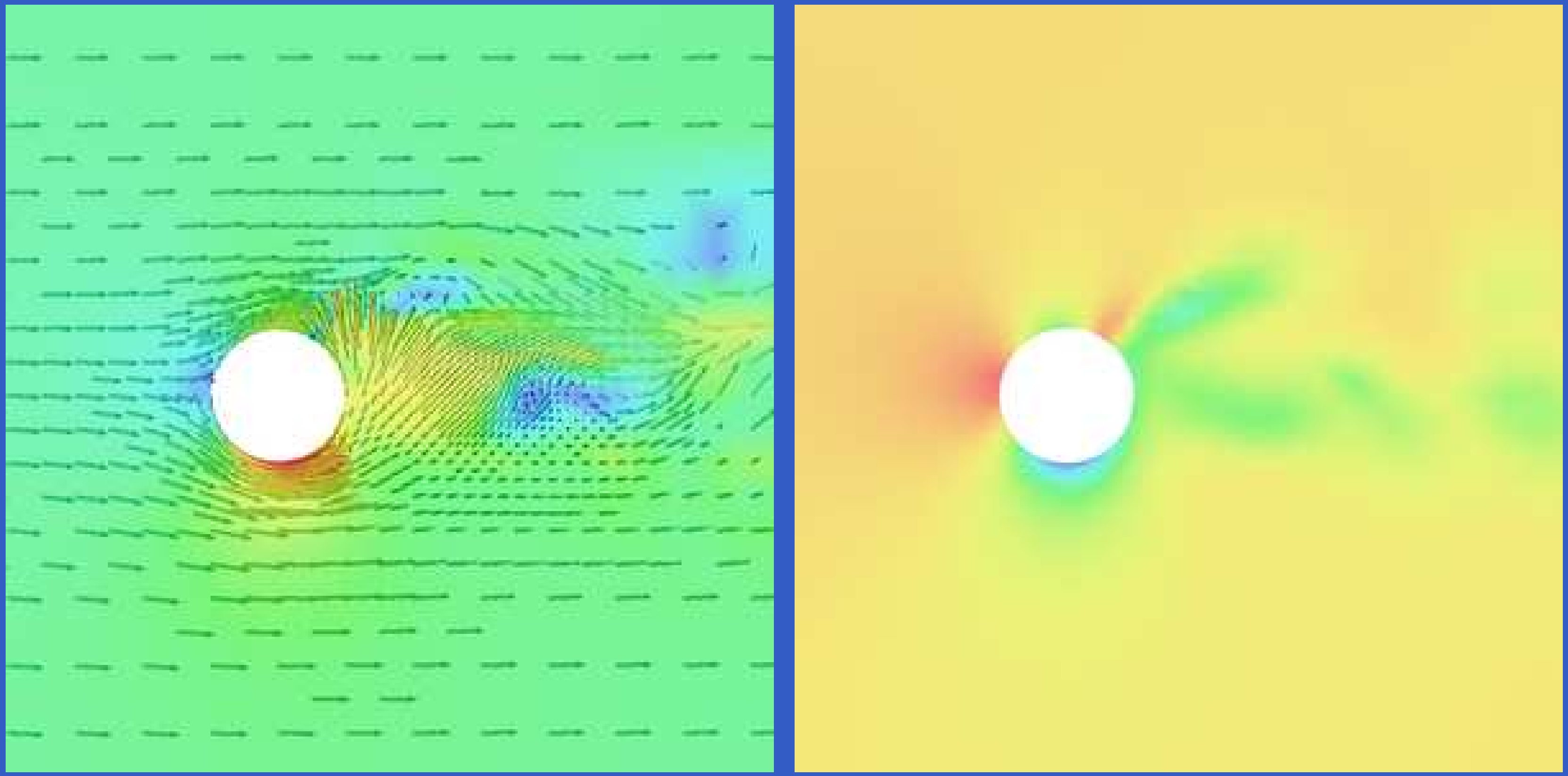




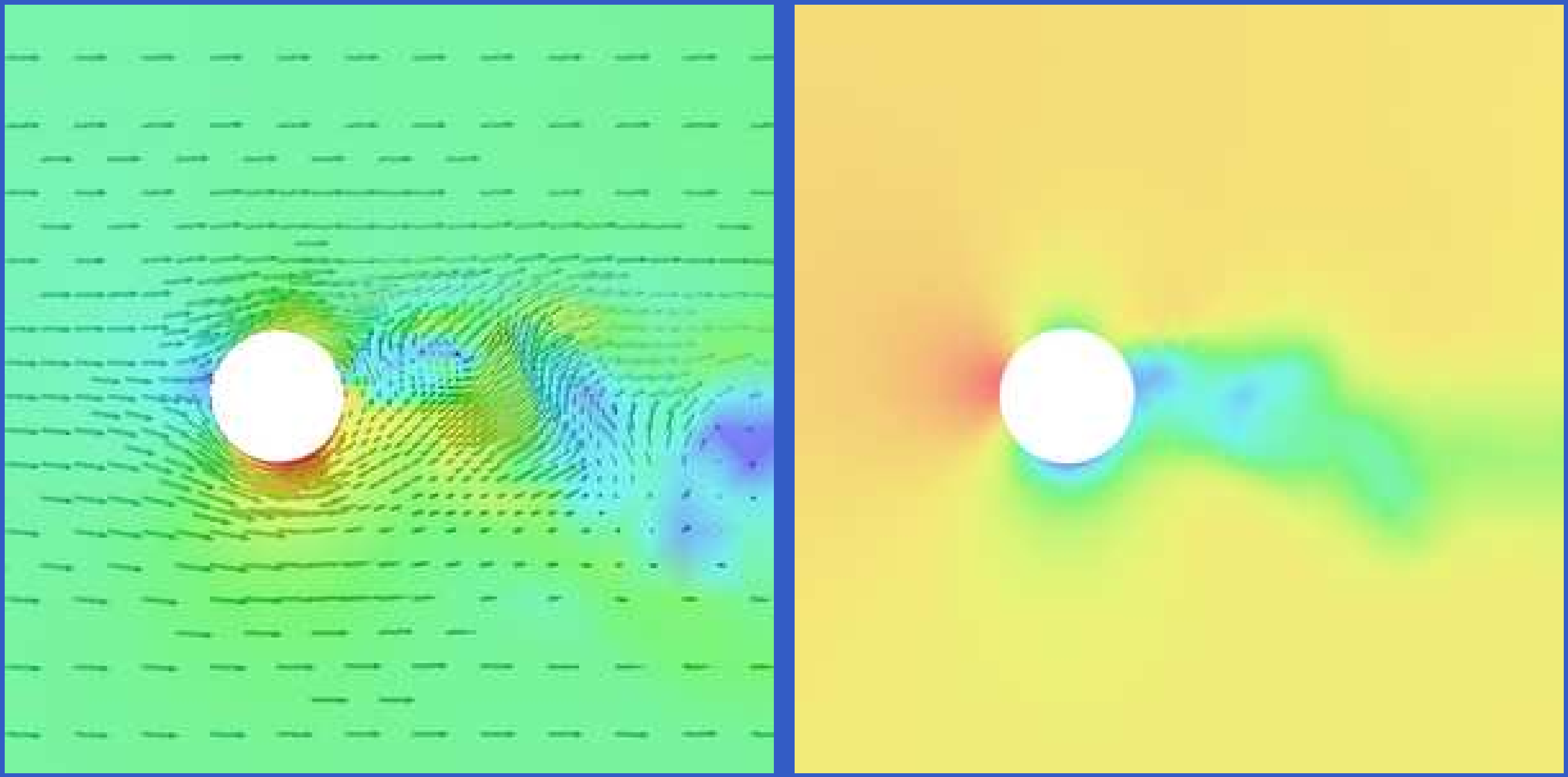
Velocity & pressure: t=2.75: $c_D = 1.04$

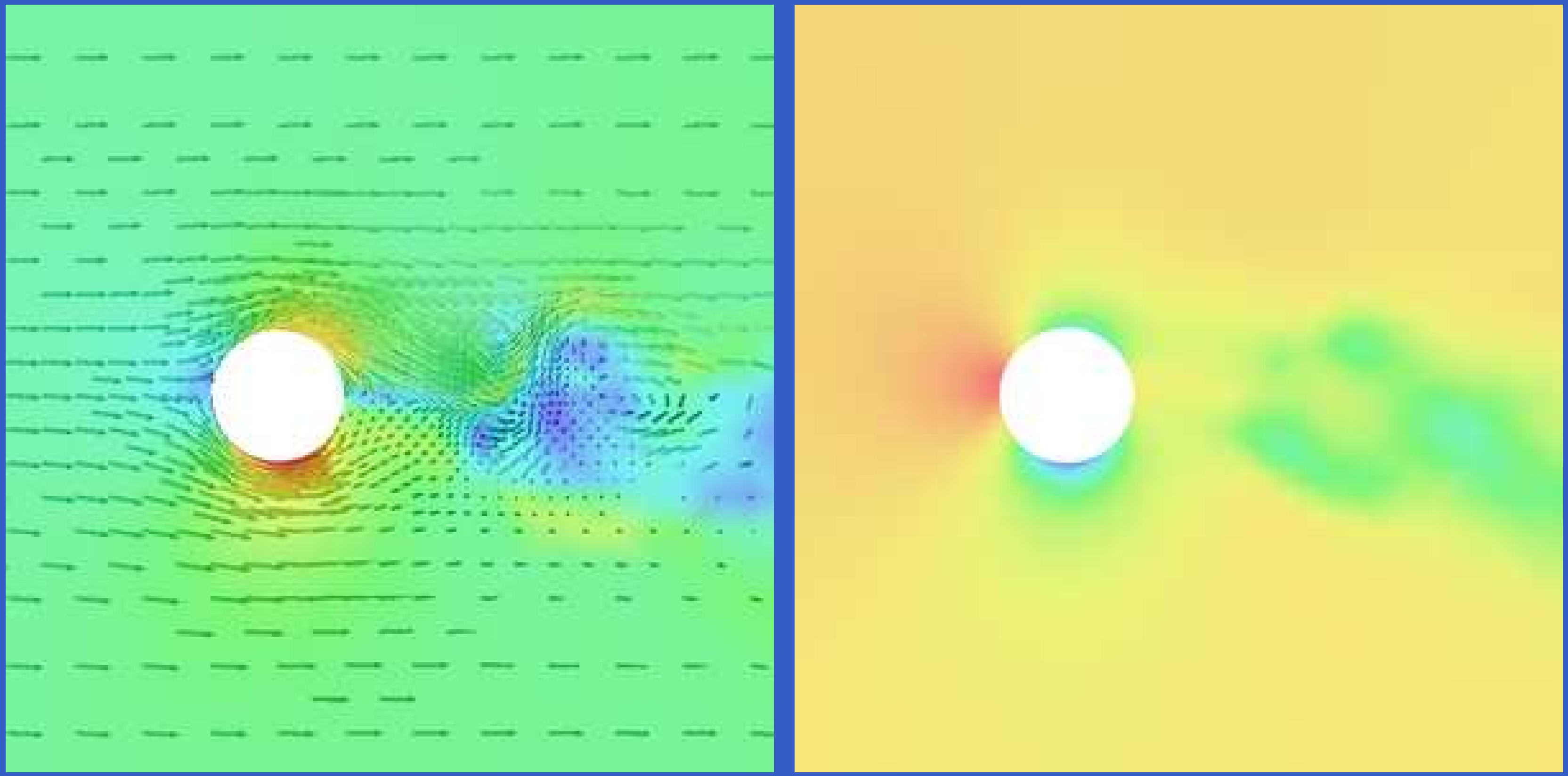
Velocity & pressure: t=3.0: $c_D = 1.05$



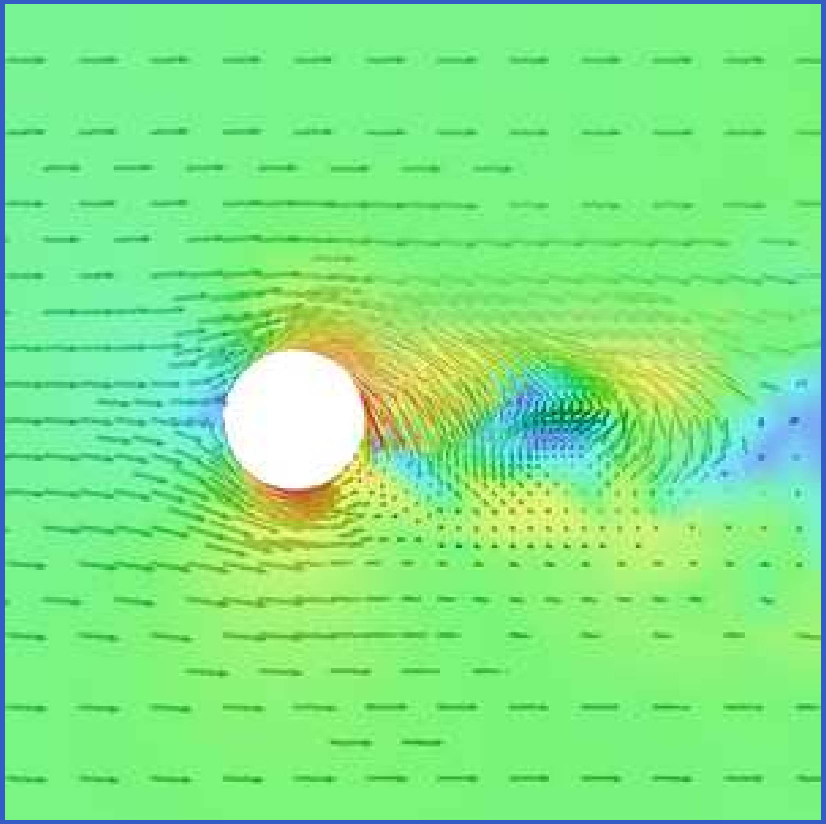
Velocity & pressure: t=4.5: $c_D = 1.63$

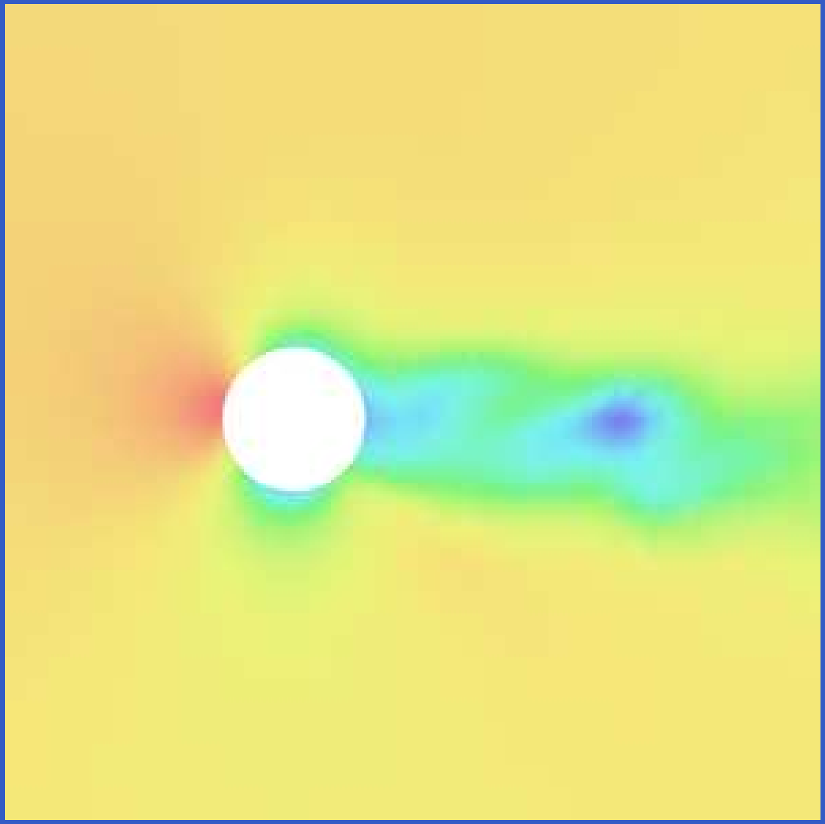


Velocity & pressure: t=5.0: $c_D = 1.79$

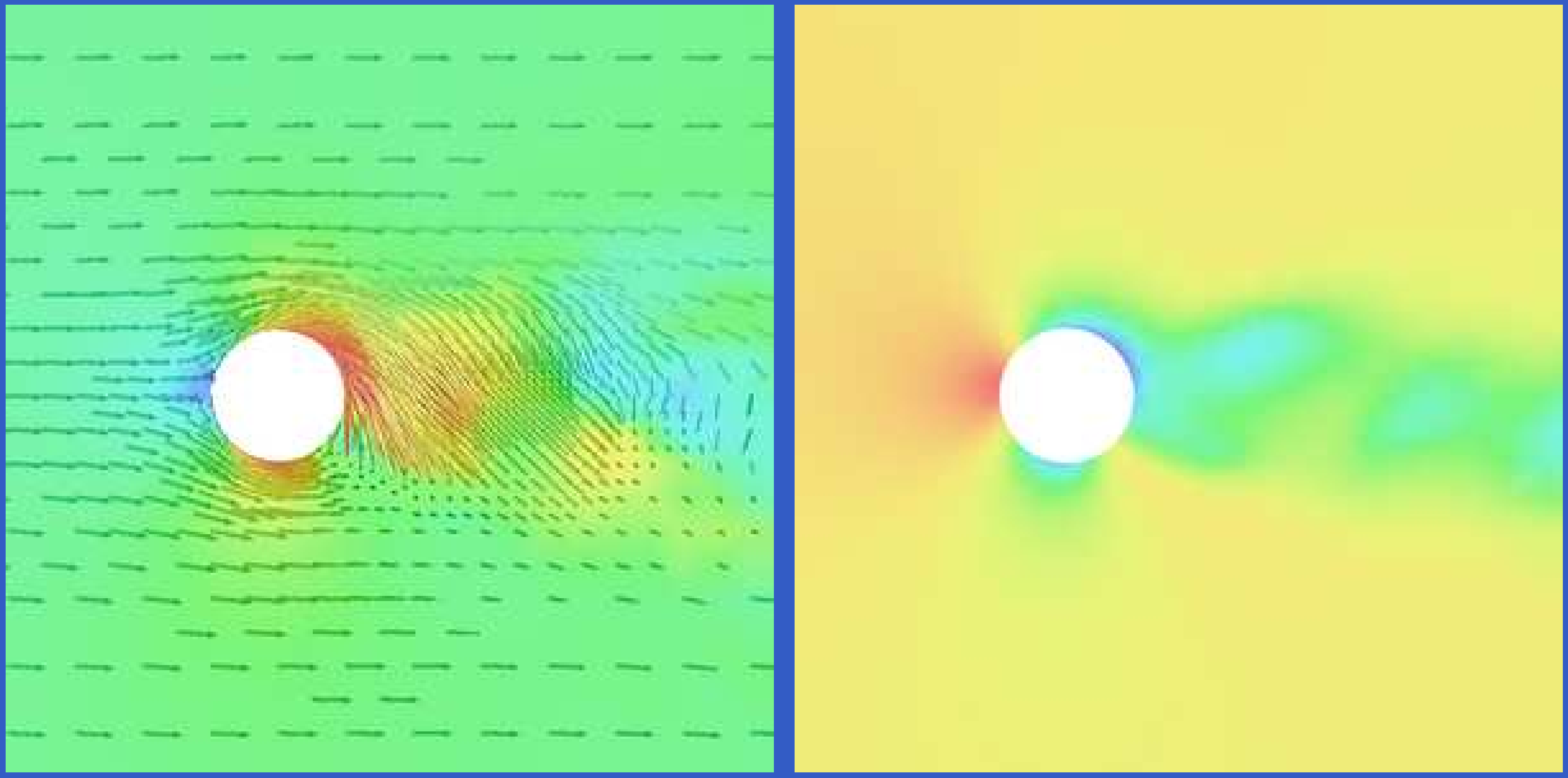


Velocity & pressure: t=5.5: $c_D = 1.96$

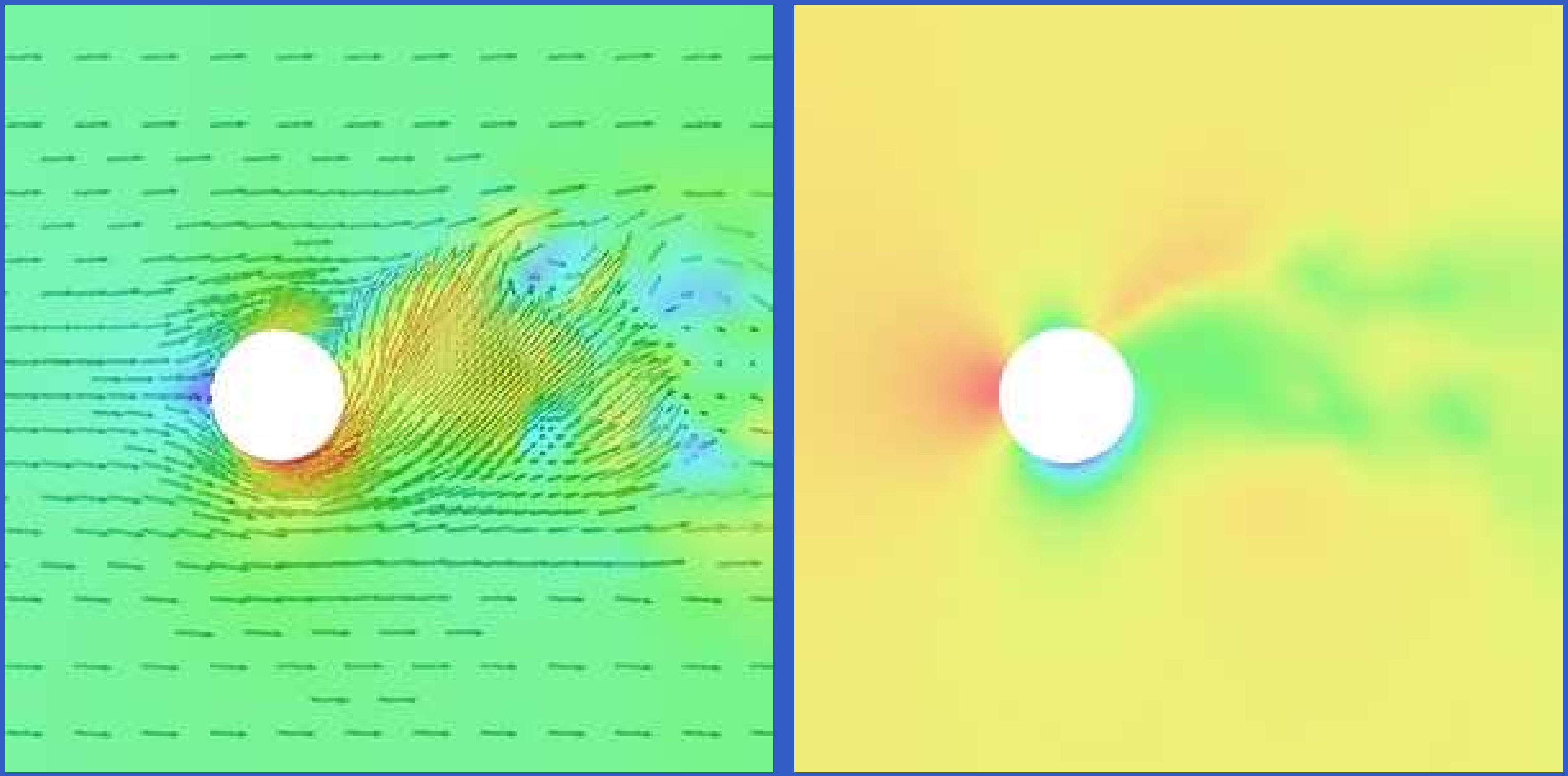




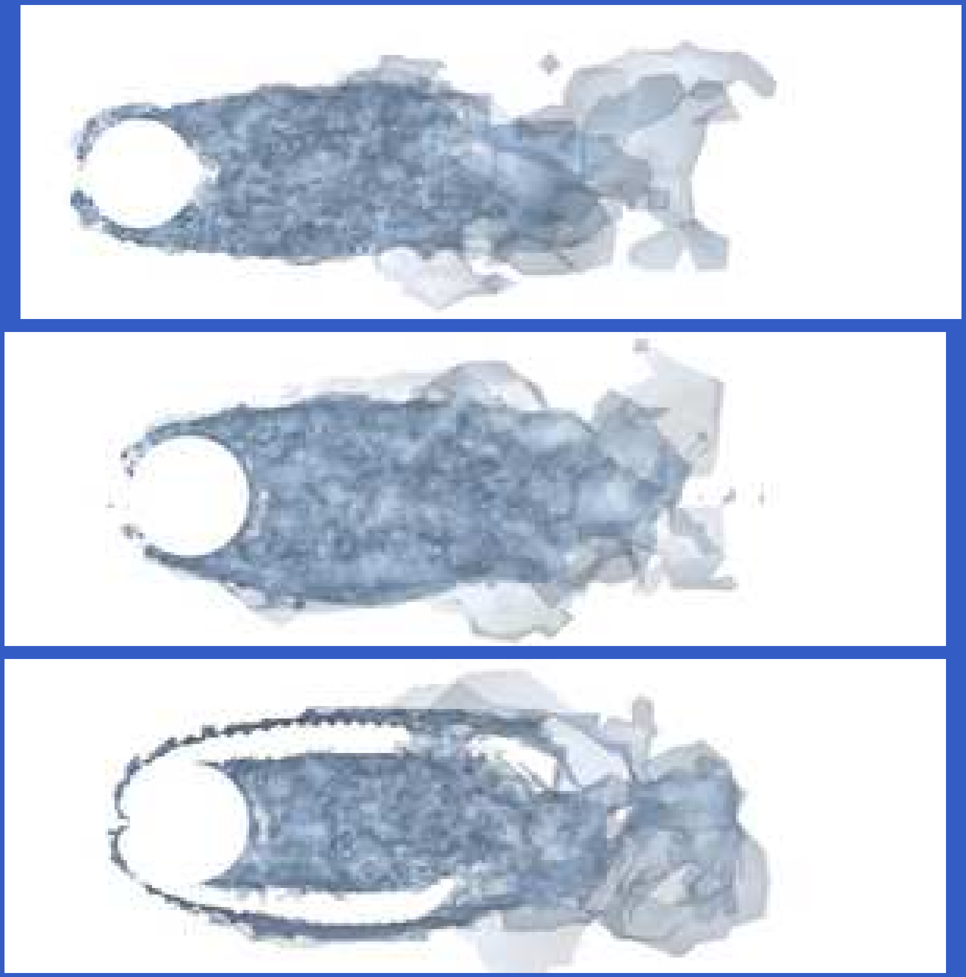
Velocity & pressure: t=5.75: $c_D = 1.90$



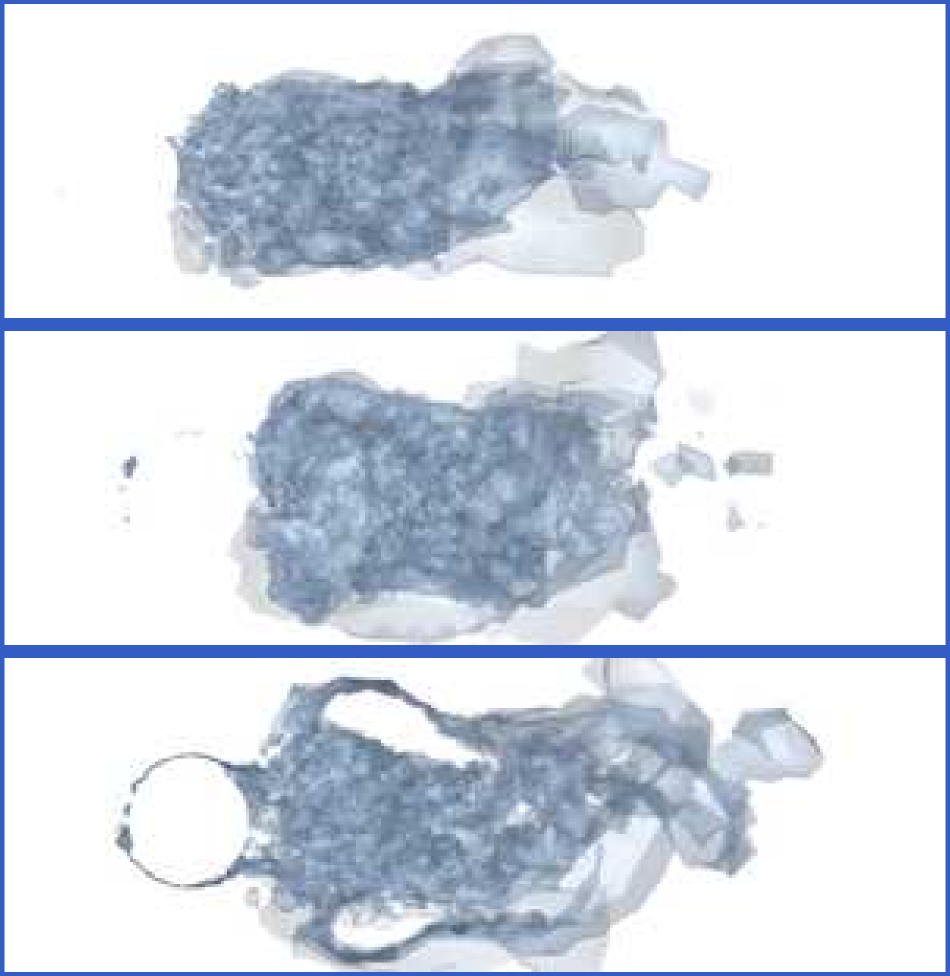
Velocity & pressure: t=11.0: $c_D = 1.82$



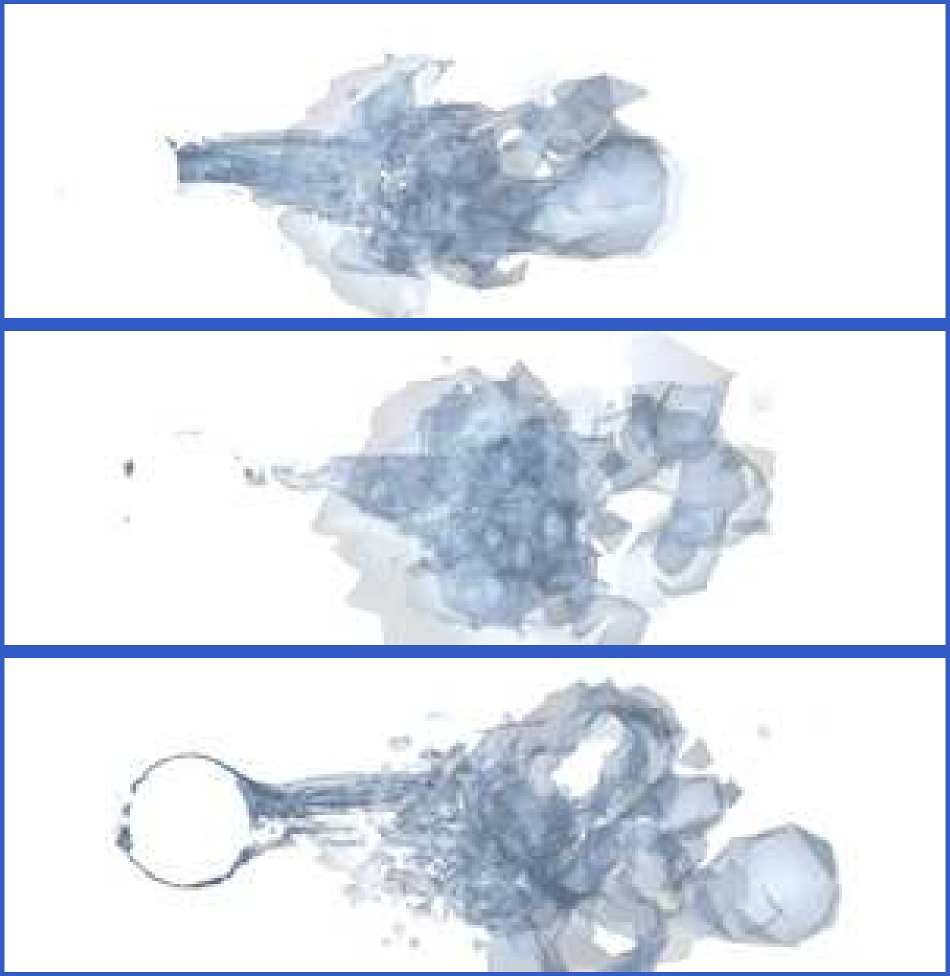
Vorticity: ω_1, ω_2, ω_3: t=0.0: $c_D = 1.03$



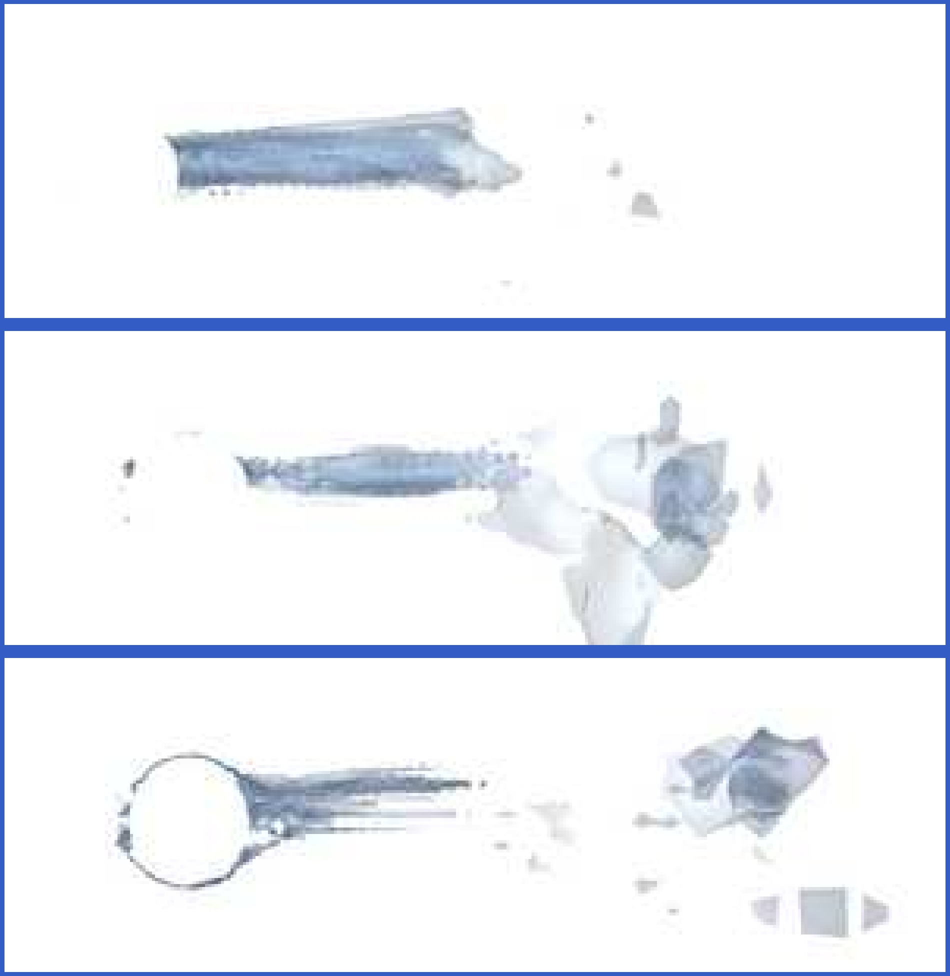
Vorticity: ω_1, ω_2, ω_3: t=0.25: $c_D = 0.06$



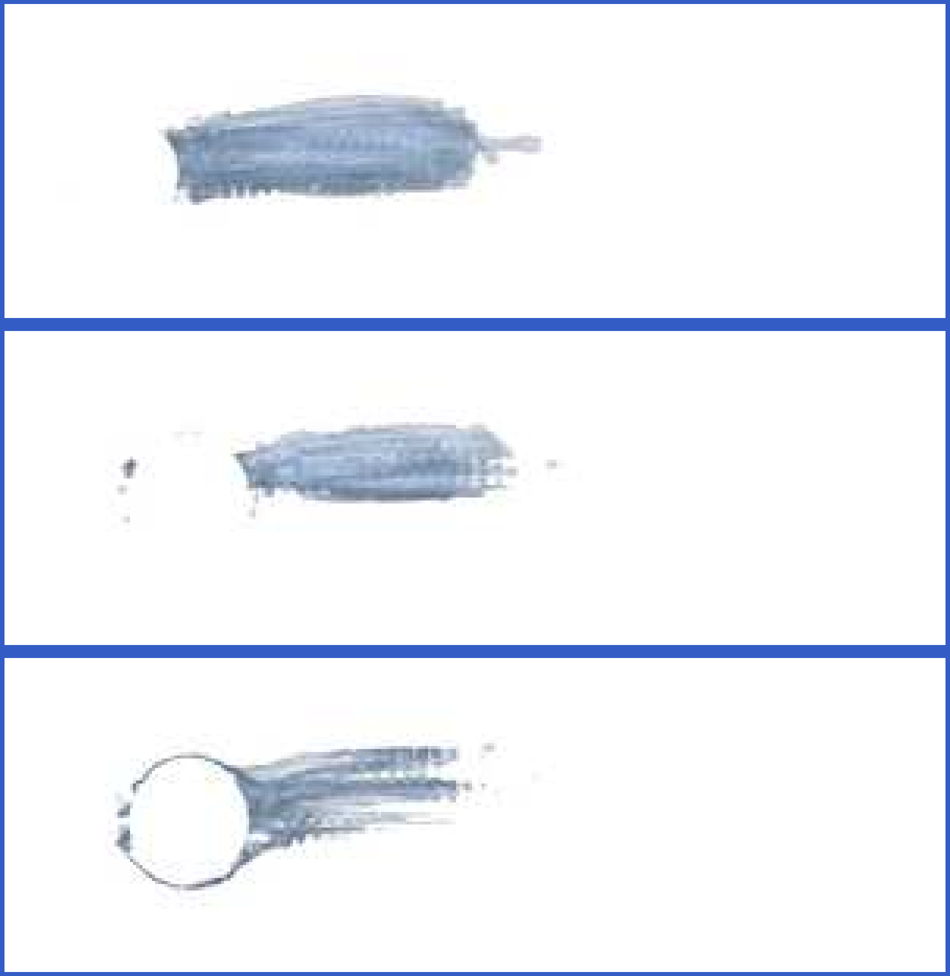
Vorticity: ω_1, ω_2, ω_3: **t=0.5:** $c_D = 0.10$



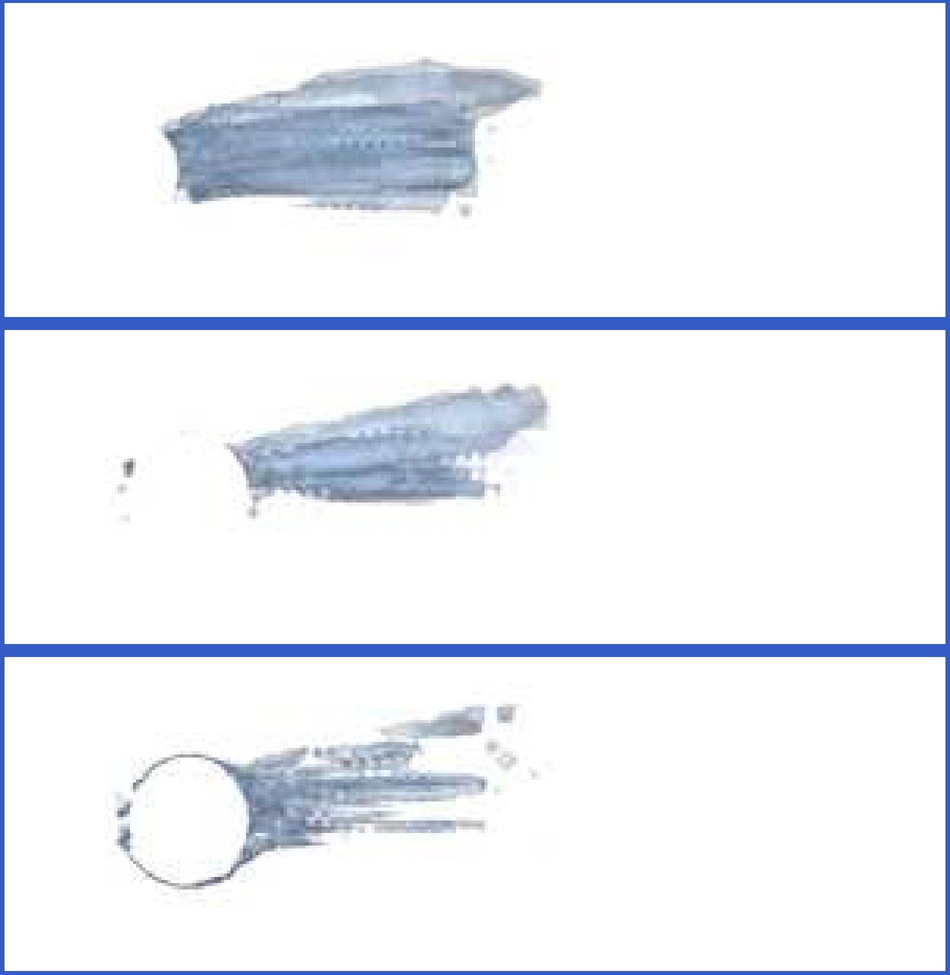
Vorticity: ω_1, ω_2, ω_3: t=0.75: $c_D = 0.15$



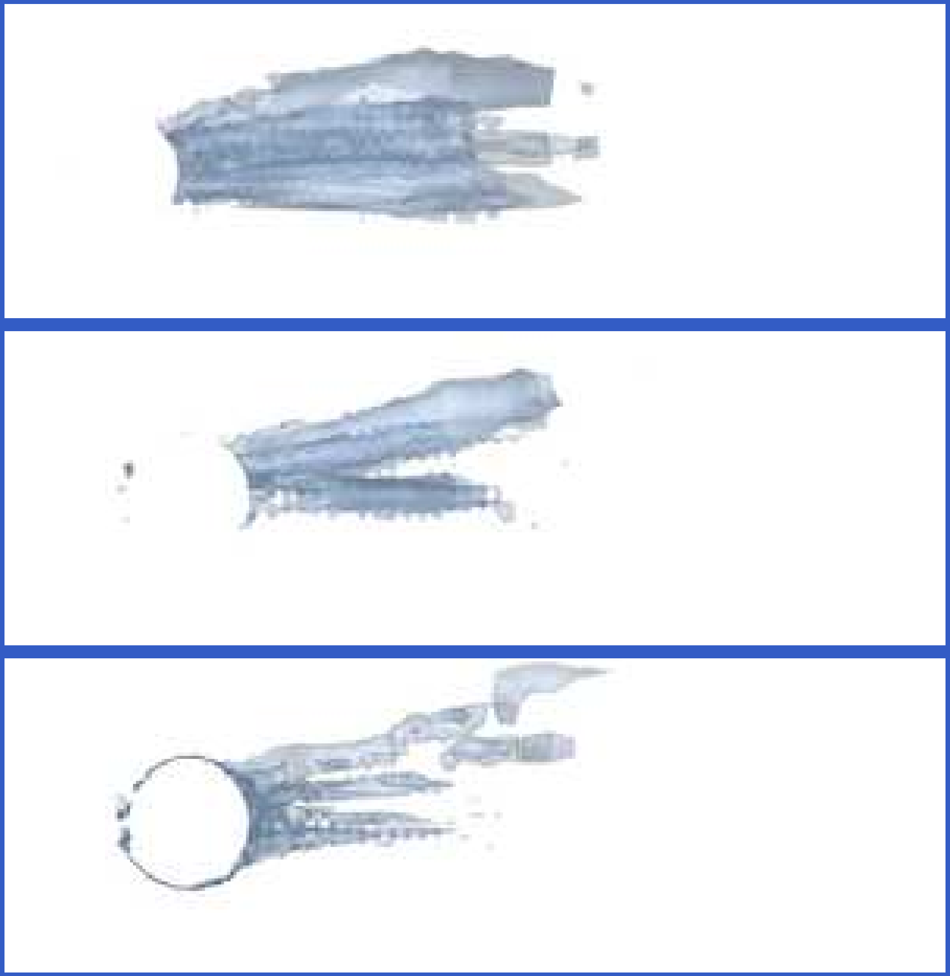
Vorticity: $\omega_1, \omega_2, \omega_3$: t=1.0: $c_D = 0.22$



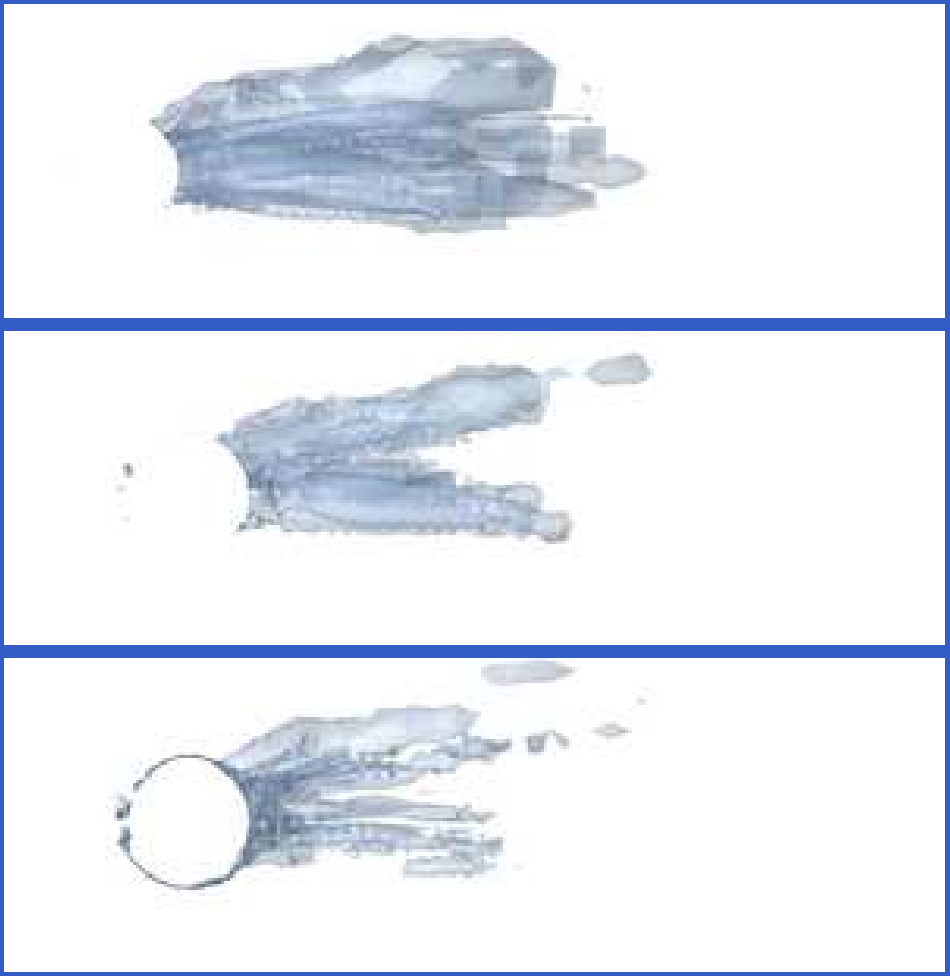
Vorticity: ω_1, ω_2, ω_3: t=1.25: $c_D = 0.25$



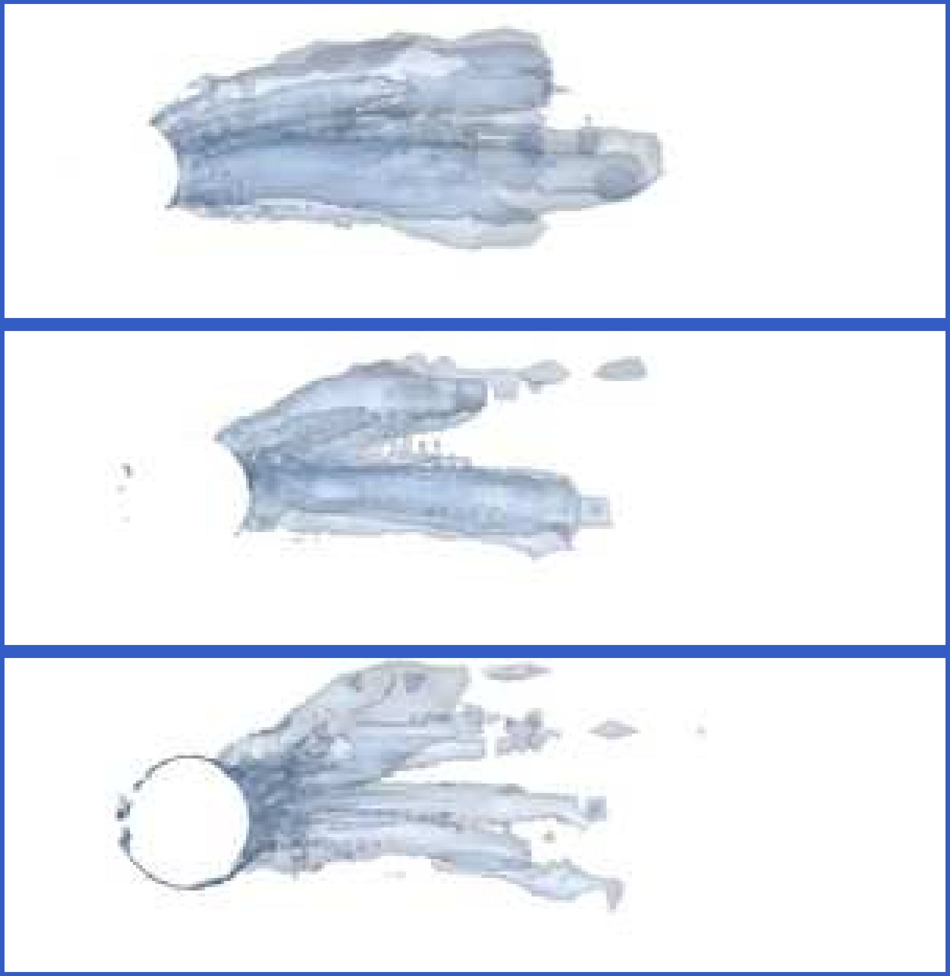
Vorticity: ω_1, ω_2, ω_3: t=1.5: $c_D = 0.28$



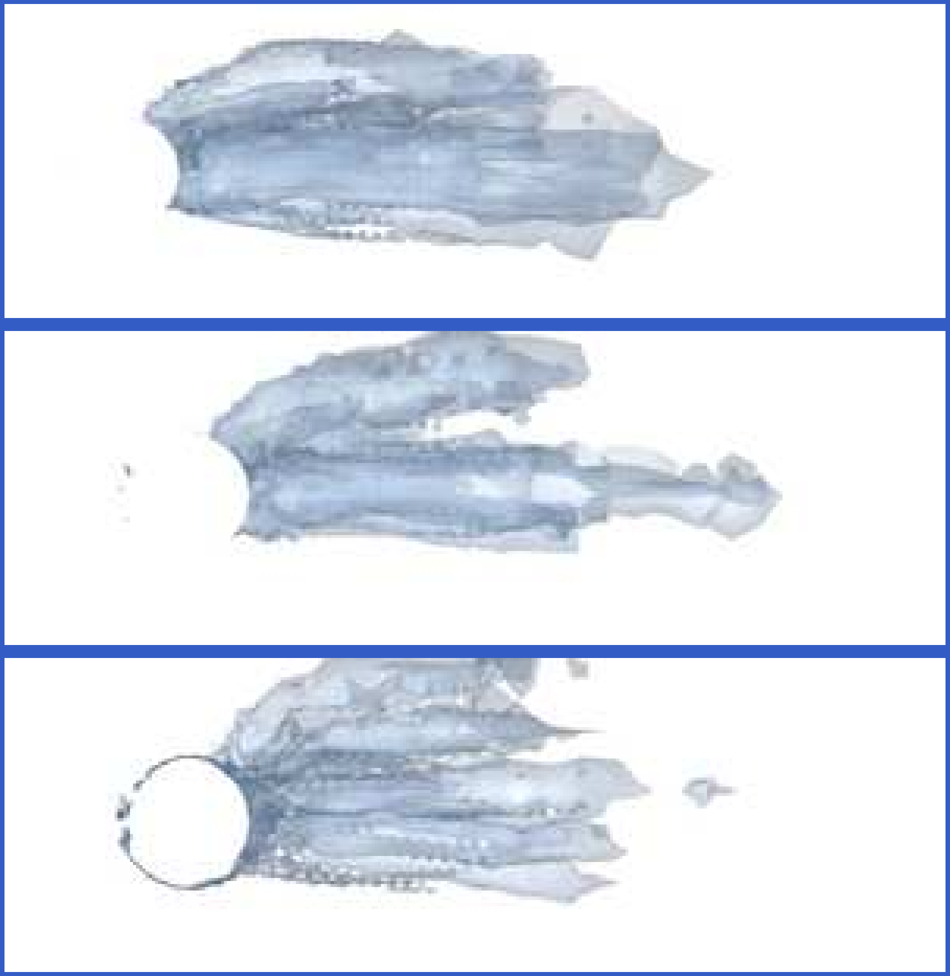
Vorticity: ω_1, ω_2, ω_3**: t=1.75:** $c_D = 0.36$



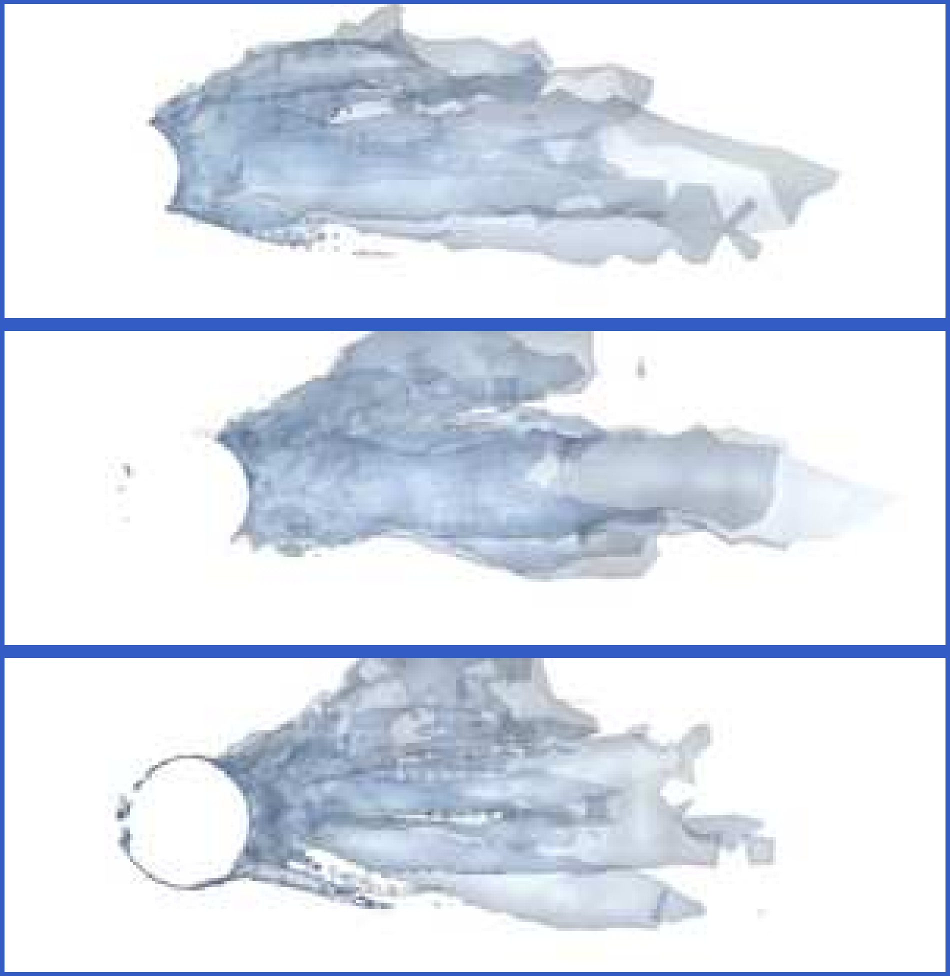
Vorticity: ω_1, ω_2, ω_3: t=2.0: $c_D = 0.51$



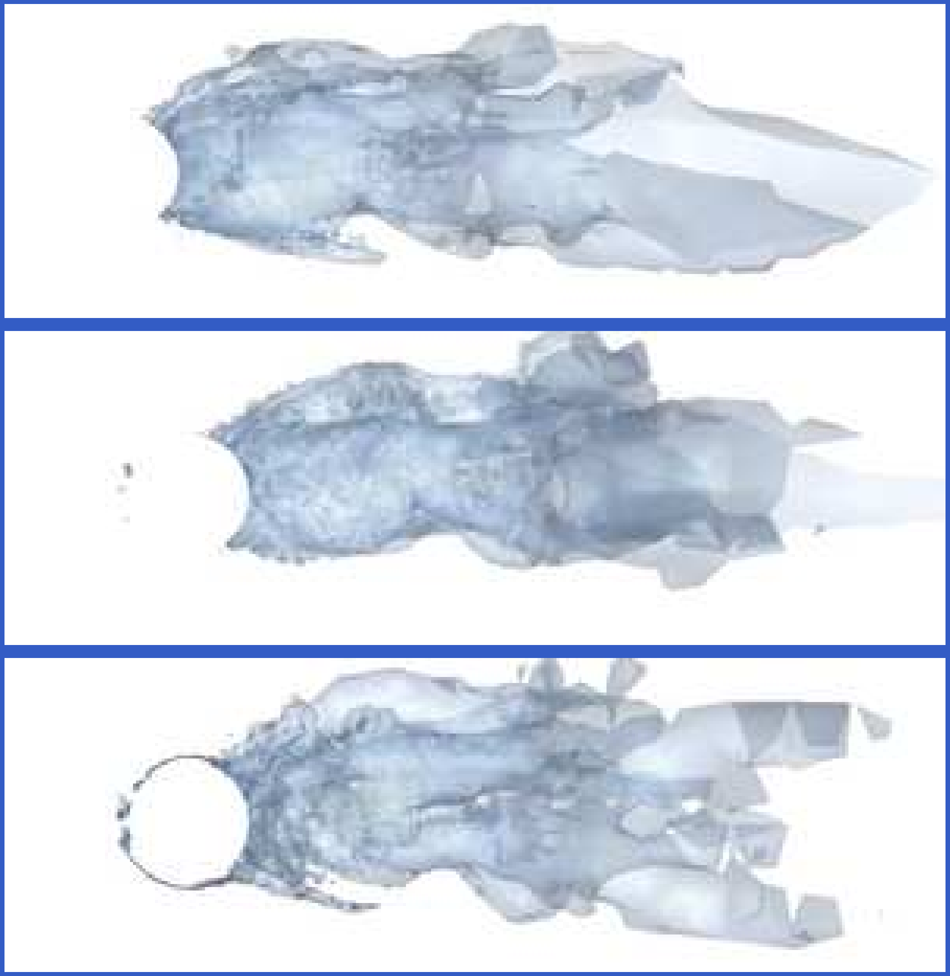
Vorticity: ω_1, ω_2, ω_3: t=2.25: $c_D = 0.78$



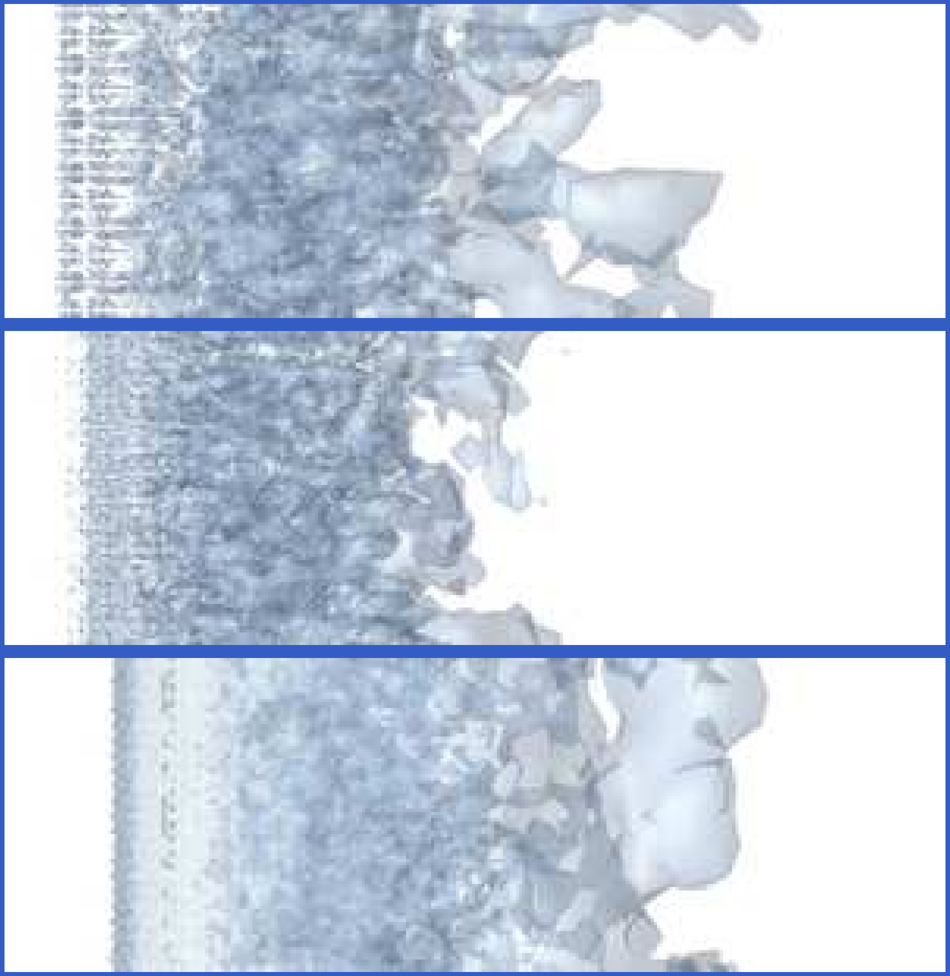
Vorticity: ω_1, ω_2, ω_3: t=2.5: $c_D = 1.14$



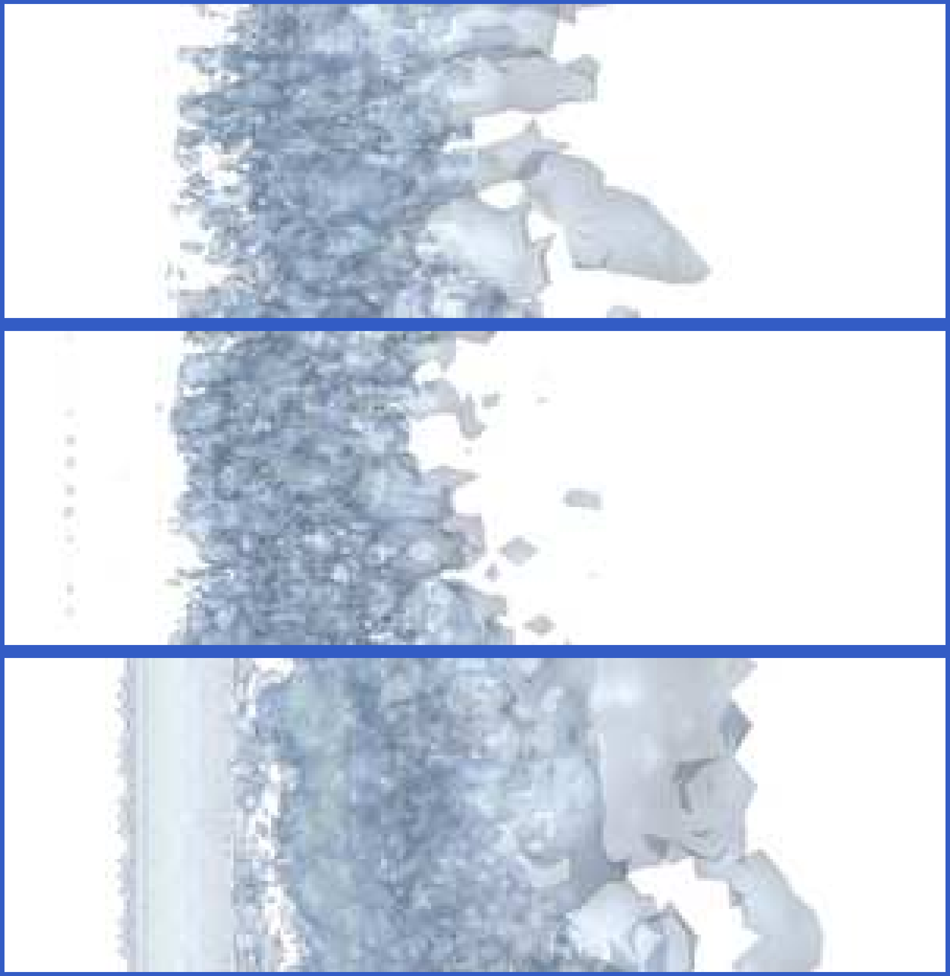
Vorticity: ω_1, ω_2, ω_3: t=2.75: $c_D = 1.04$



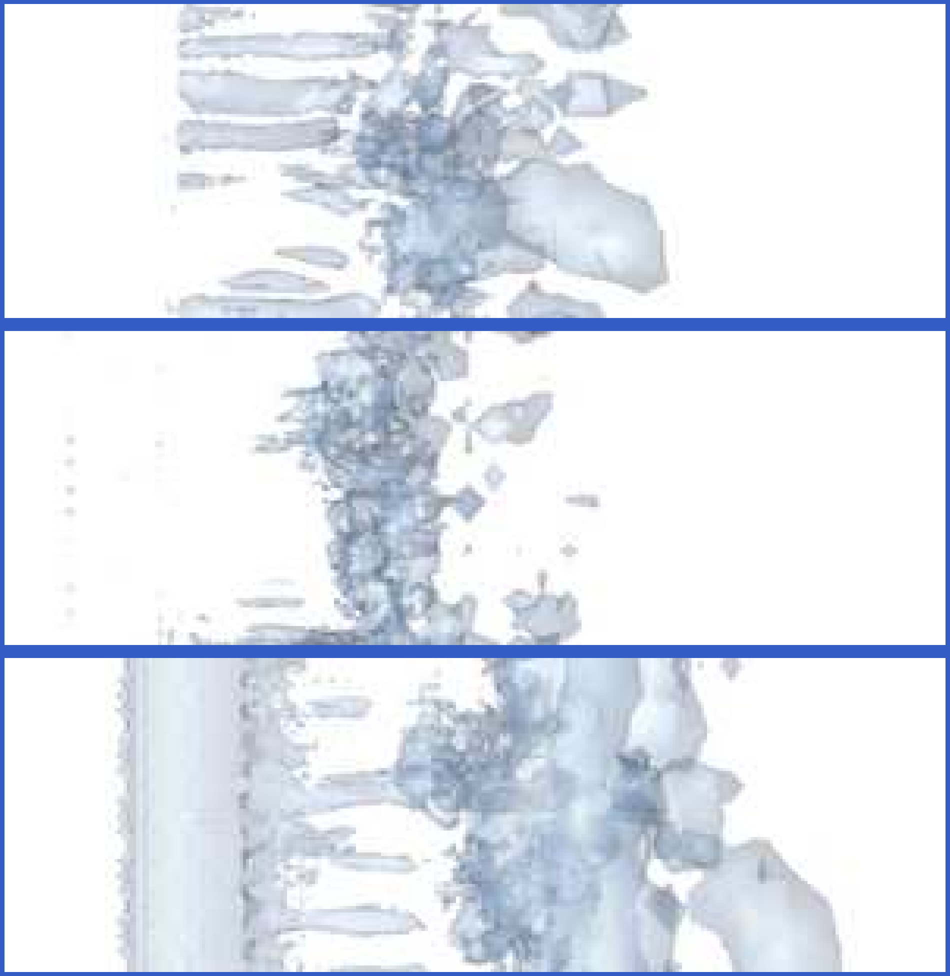
Vorticity: $\omega_1, \omega_2, \omega_3$: t=0.0: $c_D = 1.03$



Vorticity: ω_1, ω_2, ω_3: t=0.25: $c_D = 0.06$

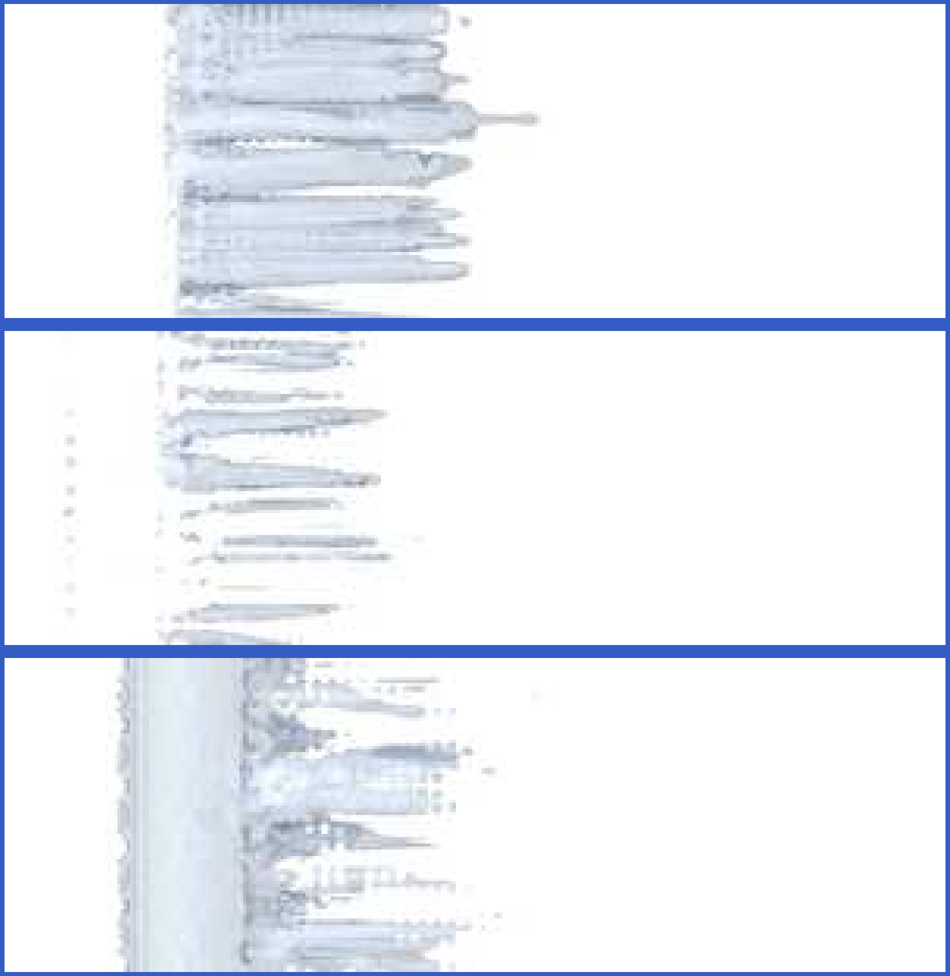


Vorticity: ω_1, ω_2, ω_3: t=0.5: $c_D = 0.10$

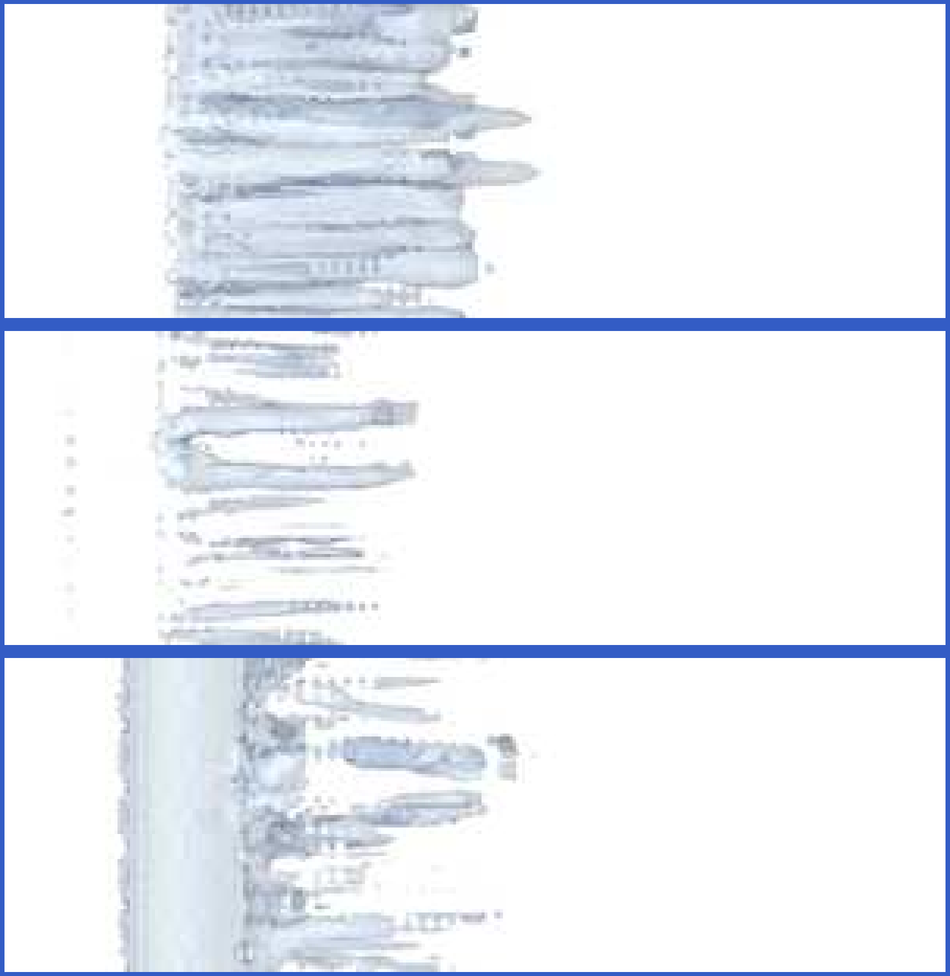


Vorticity: $\omega_1, \omega_2, \omega_3$: t=0.75: $c_D = 0.15$

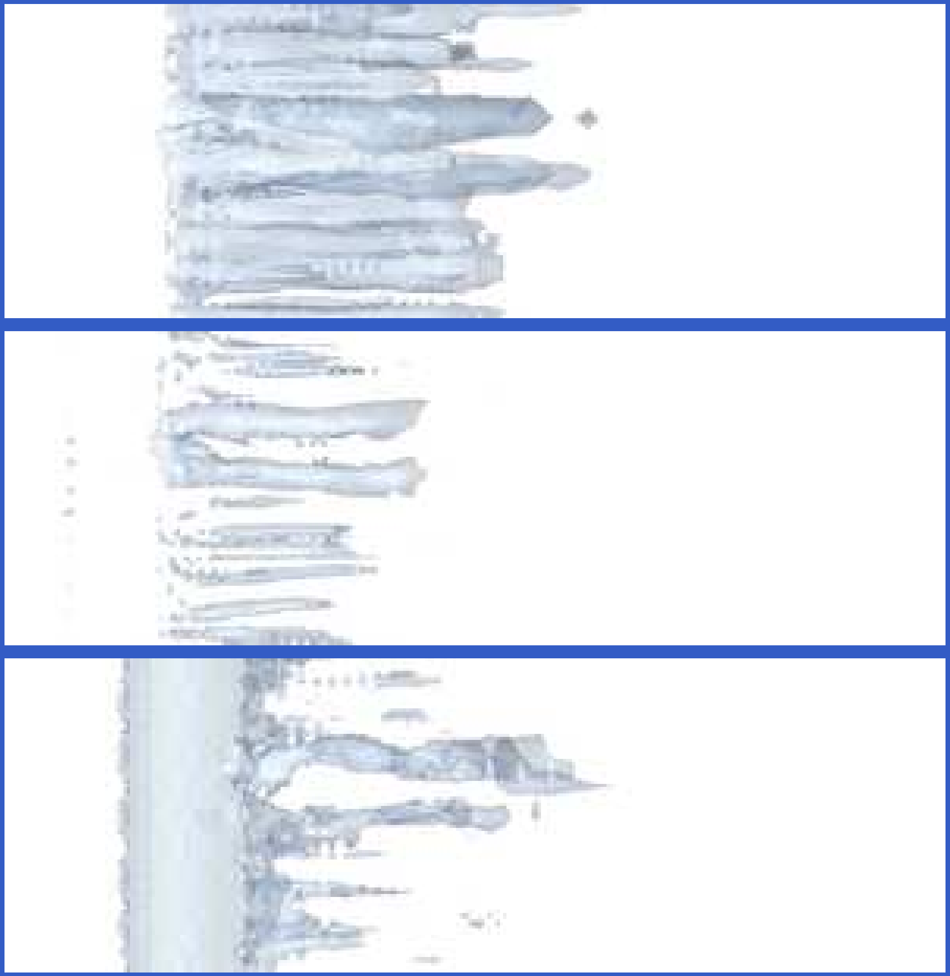
Vorticity: ω_1, ω_2, ω_3: **t=1.0:** $c_D = 0.22$



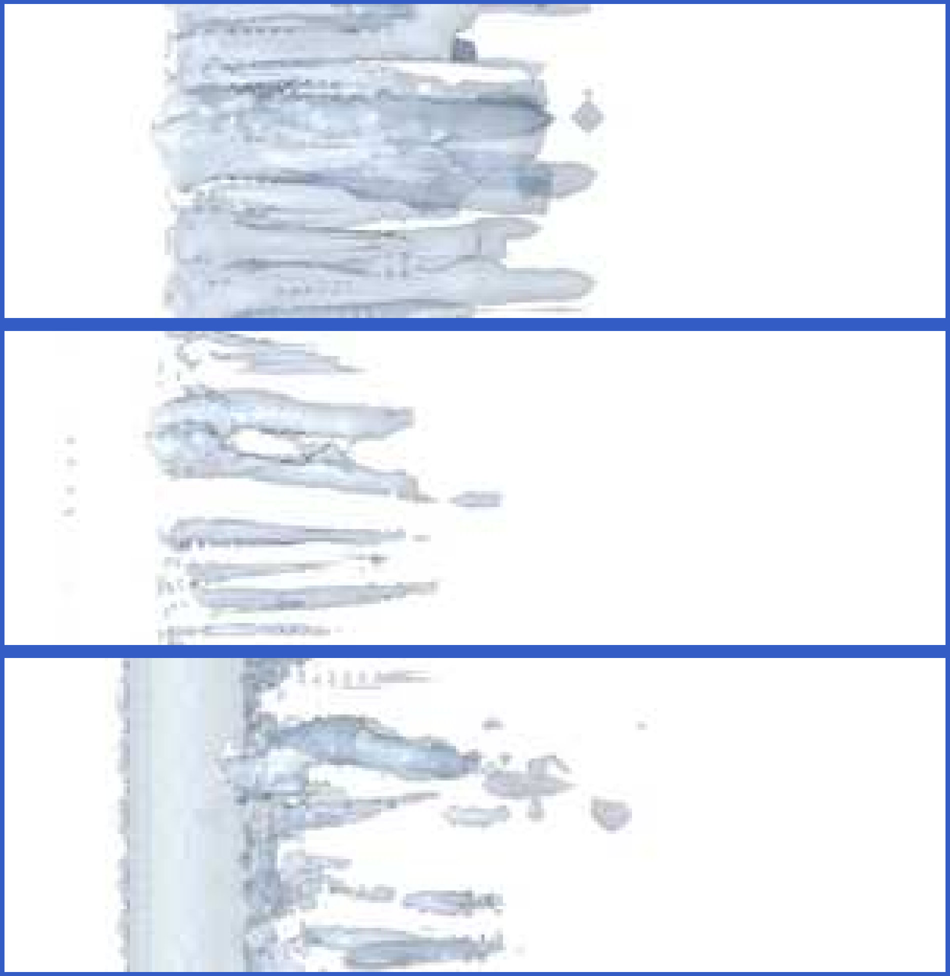
Vorticity: ω_1, ω_2, ω_3: t=1.25: $c_D = 0.25$



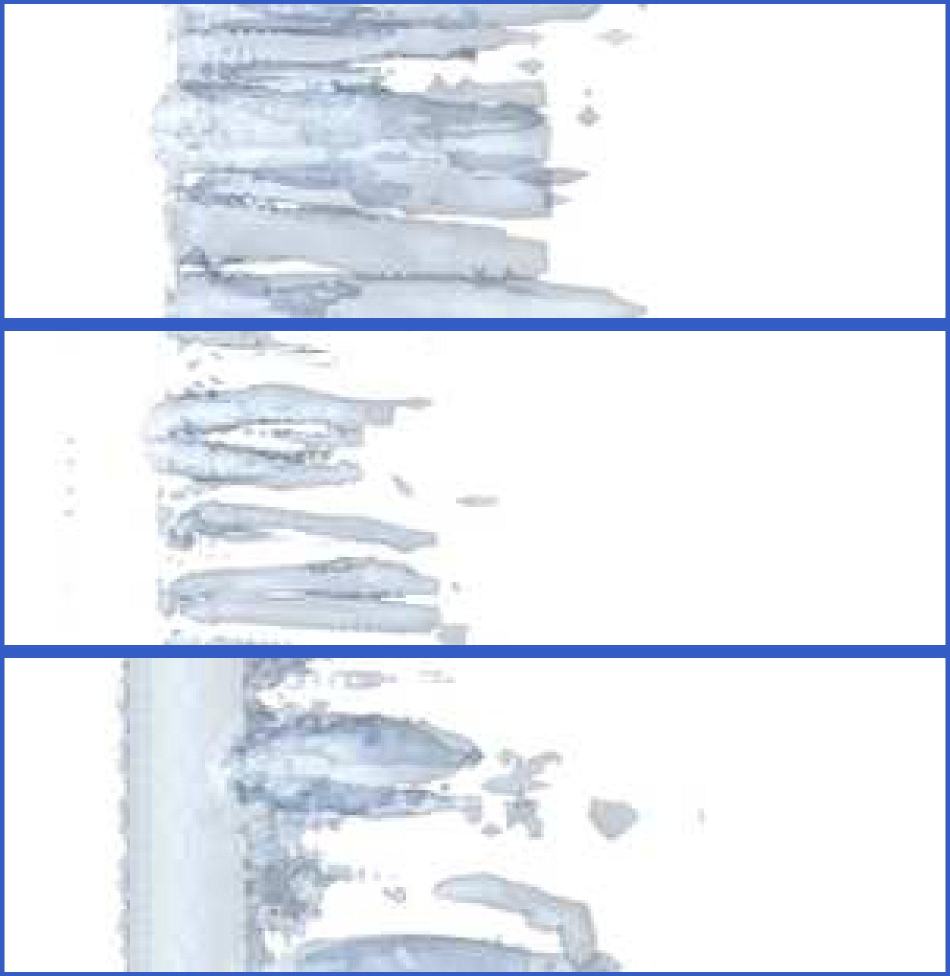
Vorticity: ω_1, ω_2, ω_3: t=1.5: $c_D = 0.28$



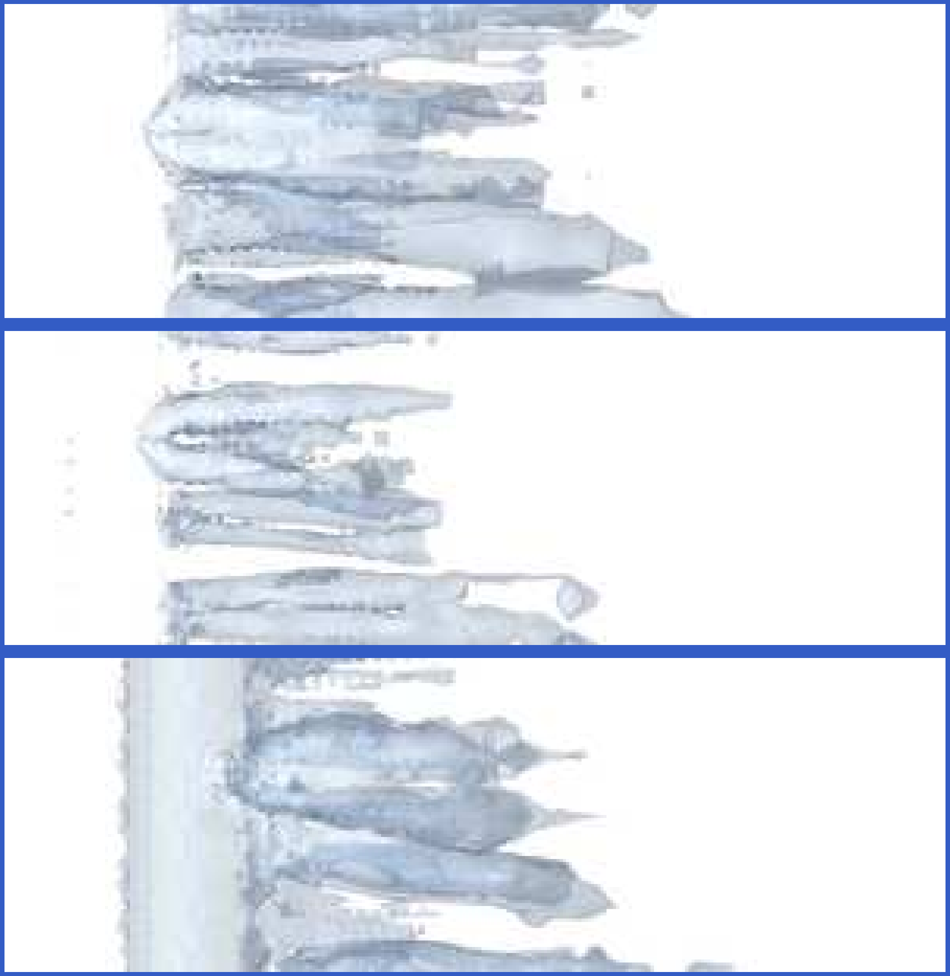
Vorticity: ω_1, ω_2, ω_3: t=1.75: $c_D = 0.36$



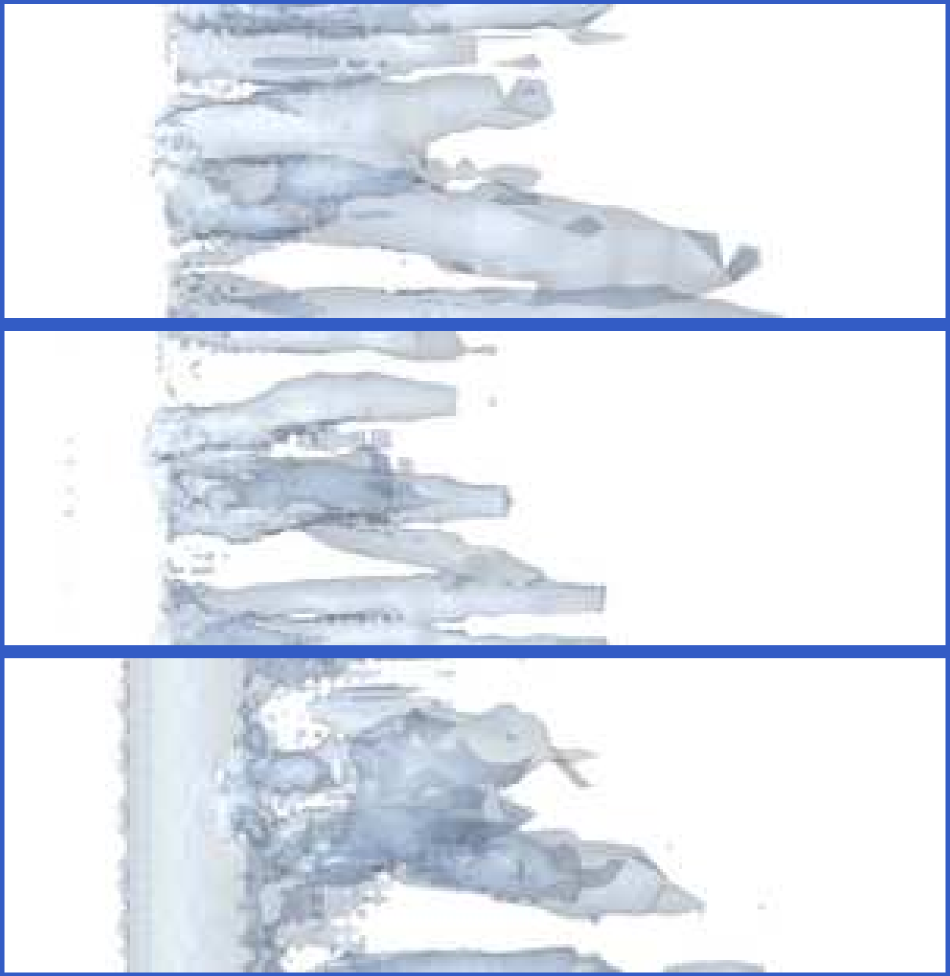
Vorticity: ω_1, ω_2, ω_3: t=2.0: $c_D = 0.51$



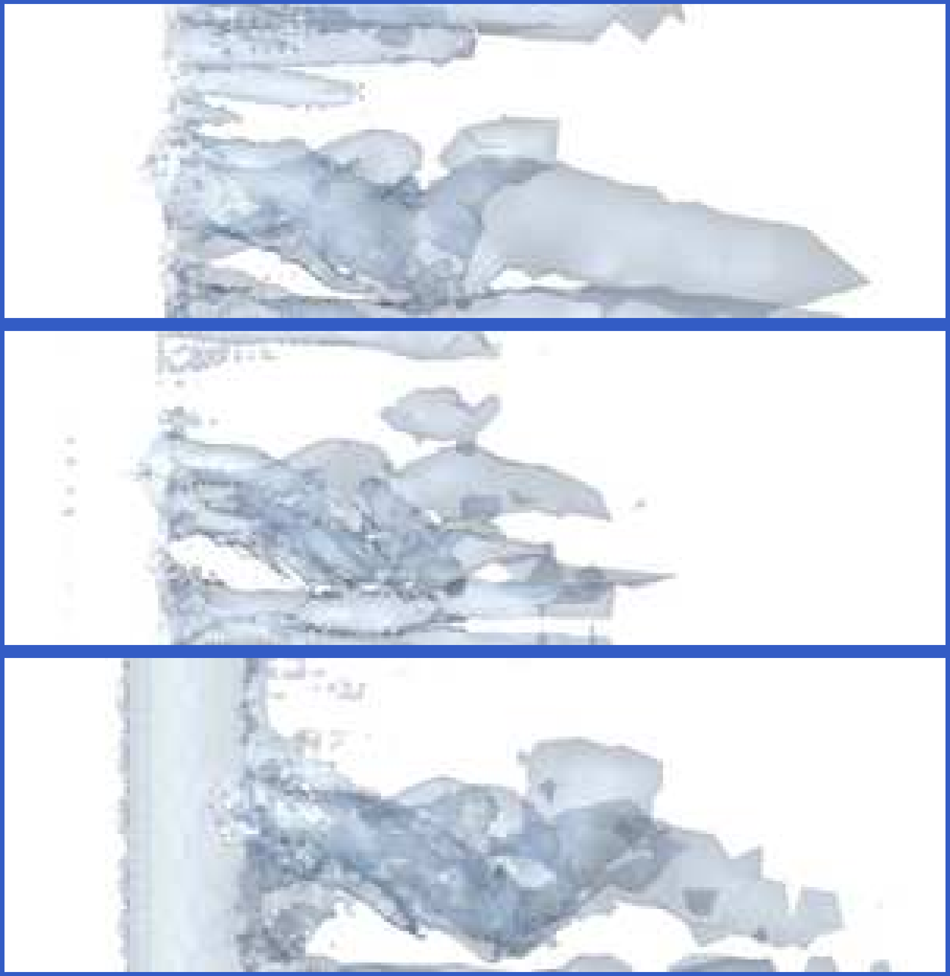
Vorticity: ω_1, ω_2, ω_3: t=2.25: $c_D = 0.78$



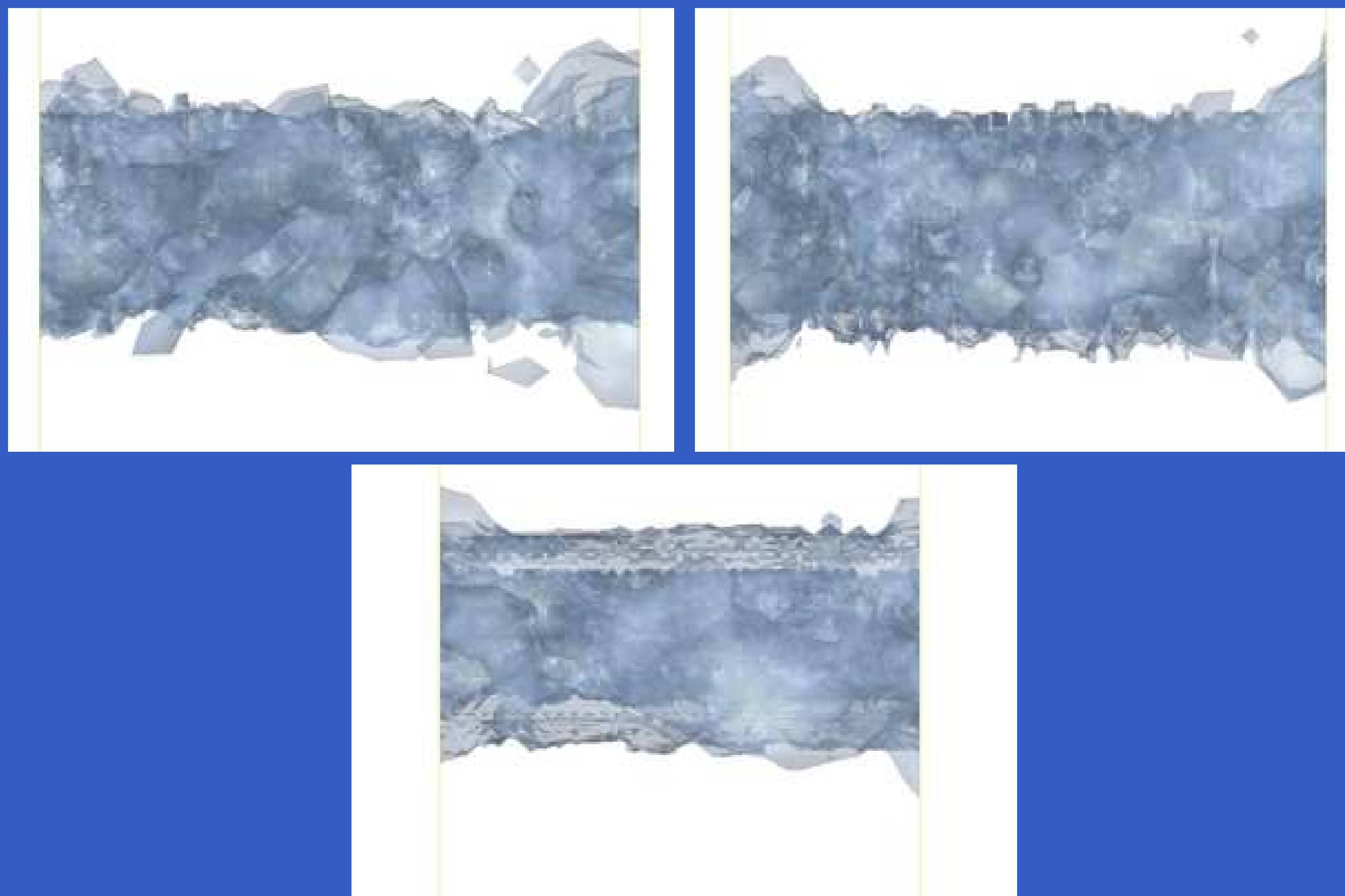
Vorticity: ω_1, ω_2, ω_3: **t=2.5:** $c_D = 1.14$



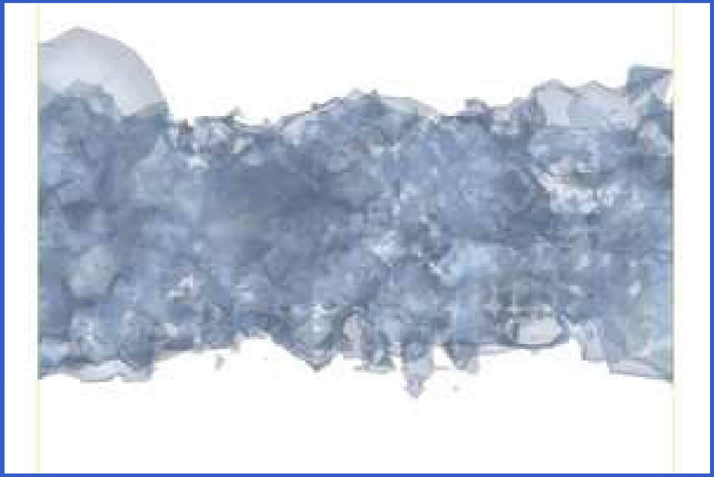
Vorticity: ω_1, ω_2, ω_3: t=2.75: $c_D = 1.04$



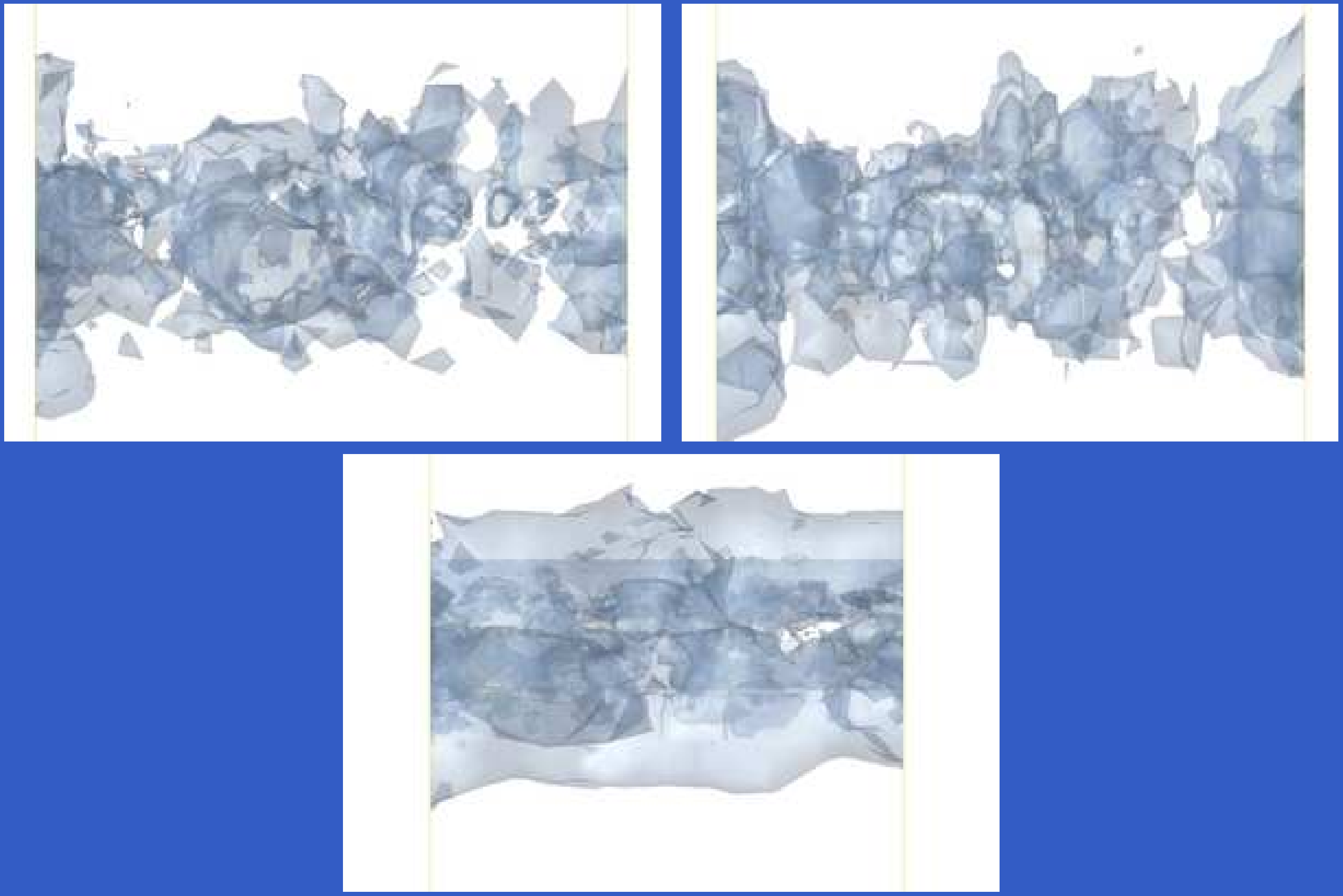
Vorticity: ω_1, ω_2, ω_3: t=0.0: $c_D = 1.03$



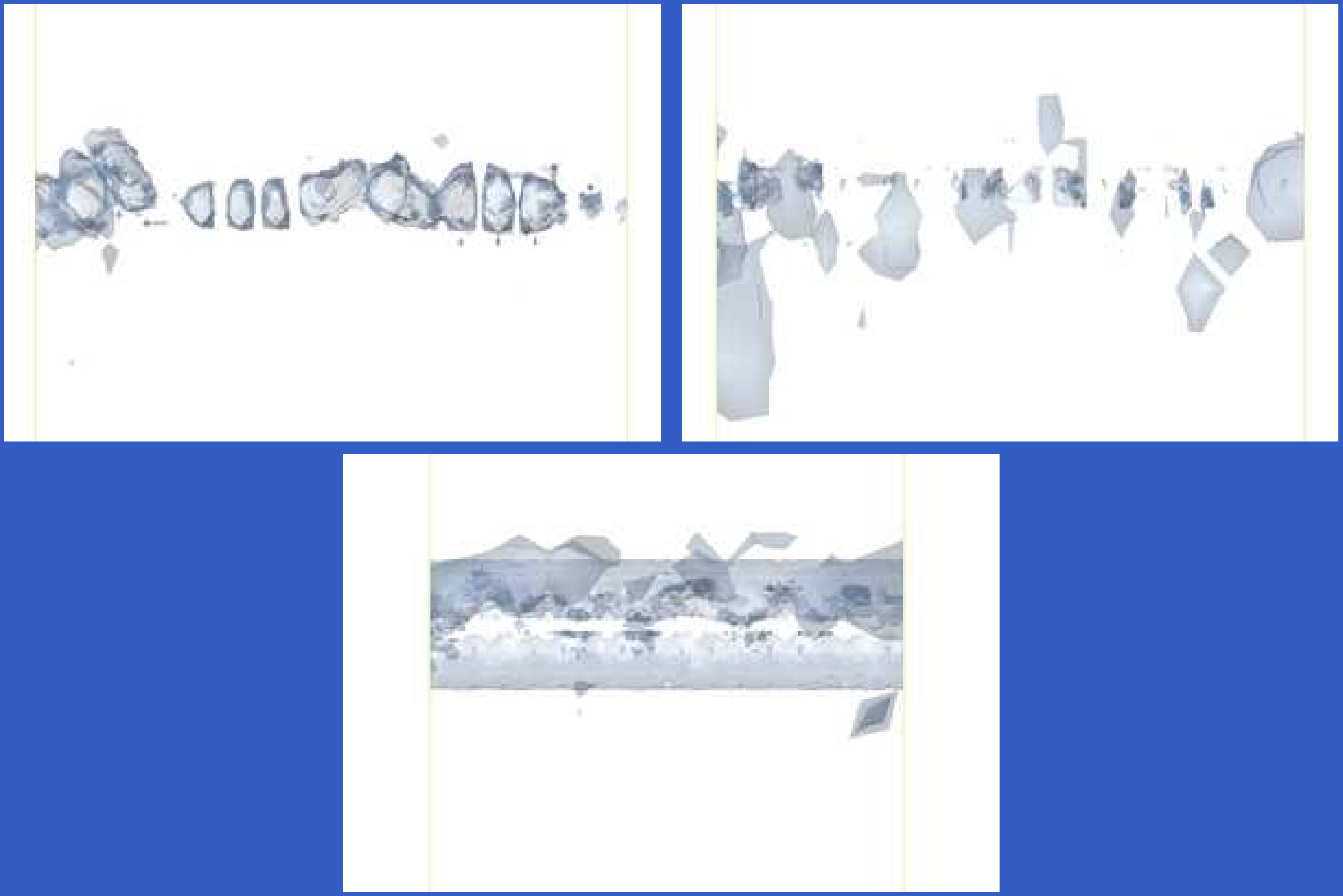
Vorticity: ω_1, ω_2, ω_3: t=0.25: $c_D = 0.06$



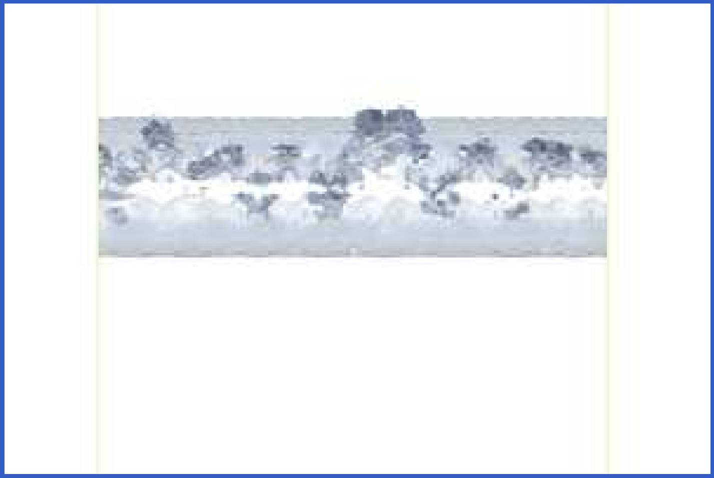
Vorticity: ω_1, ω_2, ω_3: t=0.5: $c_D = 0.10$



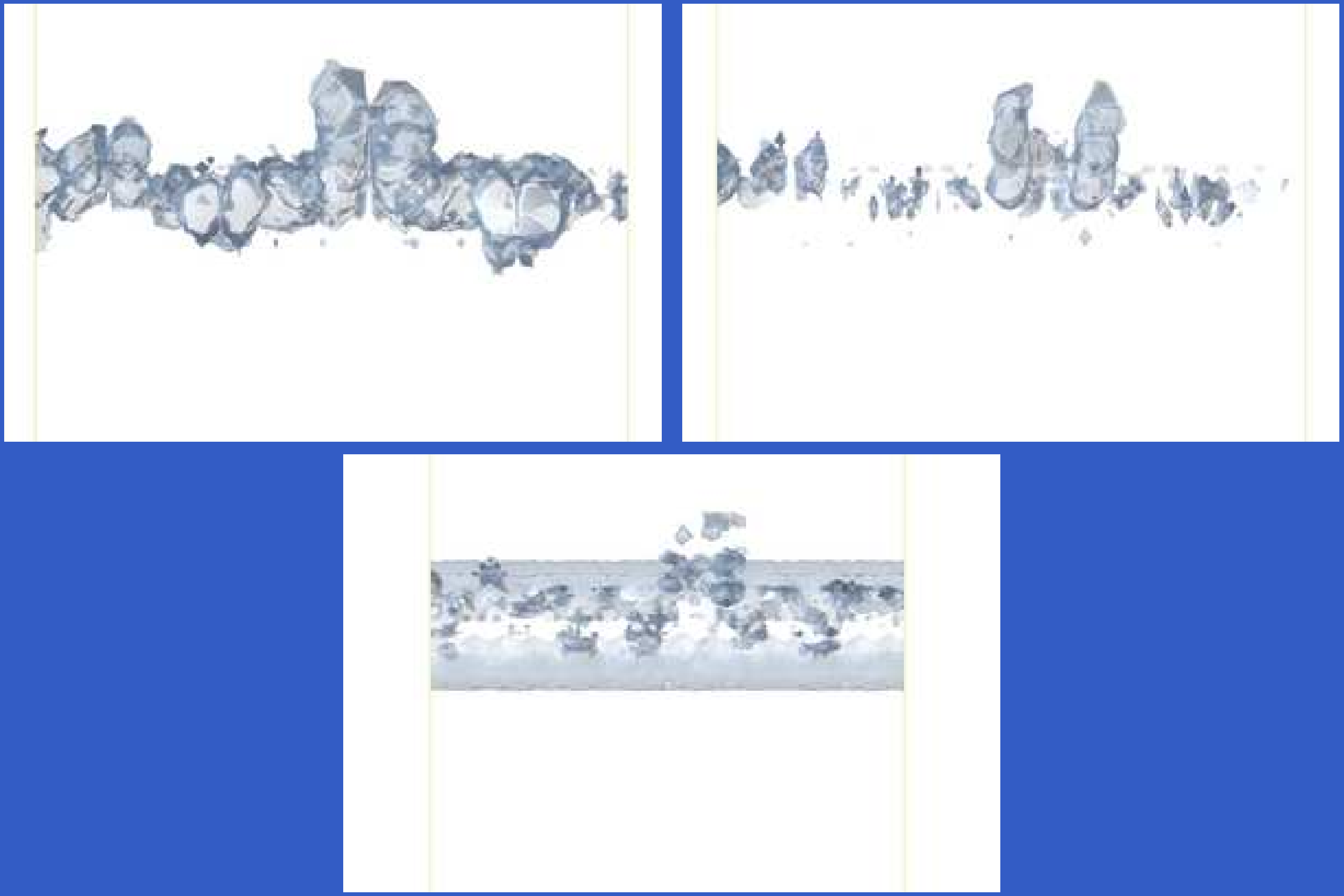
Vorticity: ω_1, ω_2, ω_3: t=0.75: $c_D = 0.15$



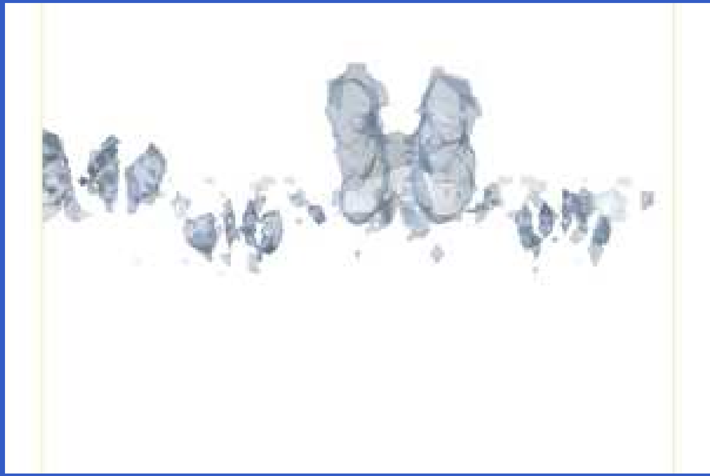
Vorticity: ω_1, ω_2, ω_3: t=1.0: $c_D = 0.22$



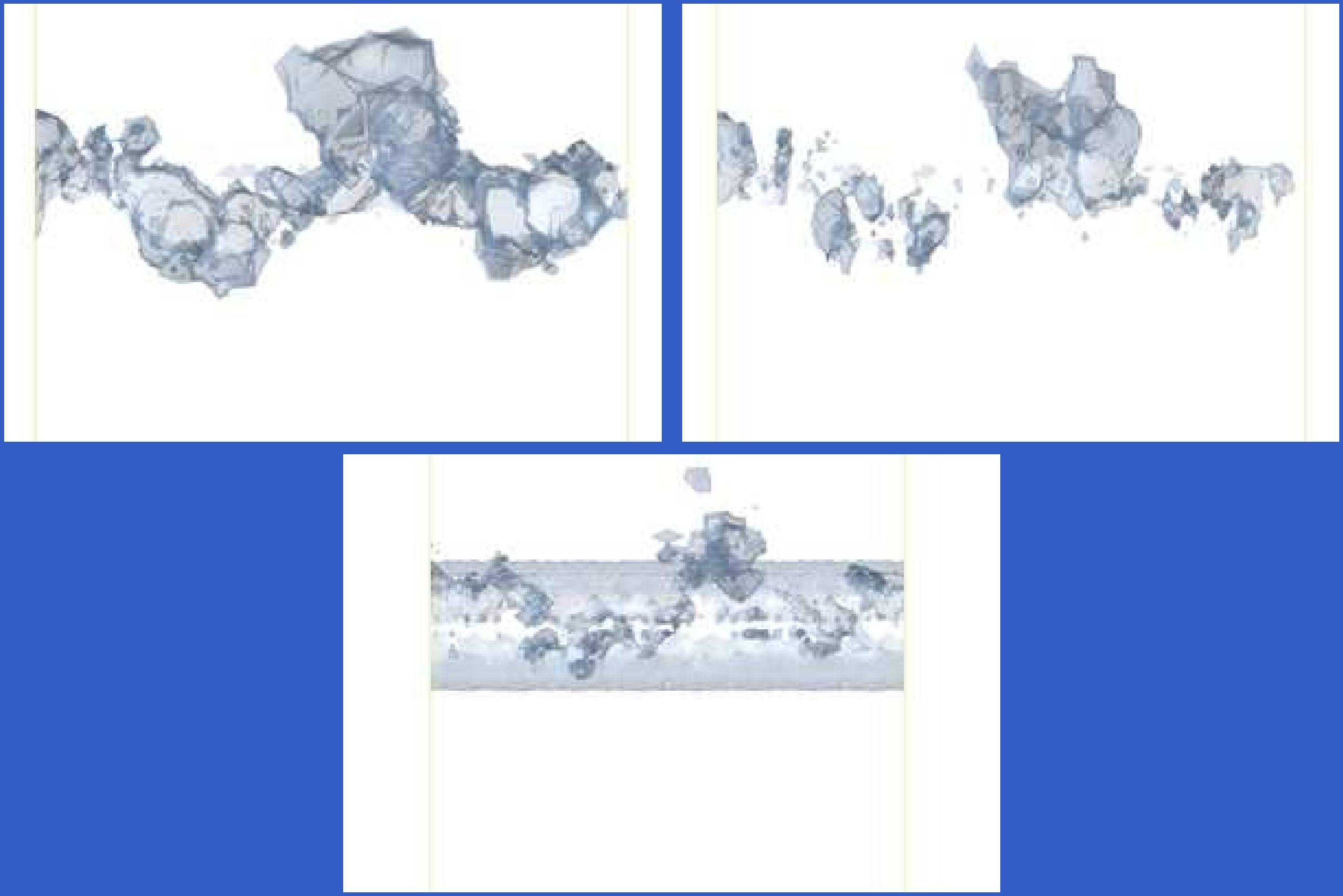
Vorticity: ω_1, ω_2, ω_3: t=1.25: $c_D = 0.25$



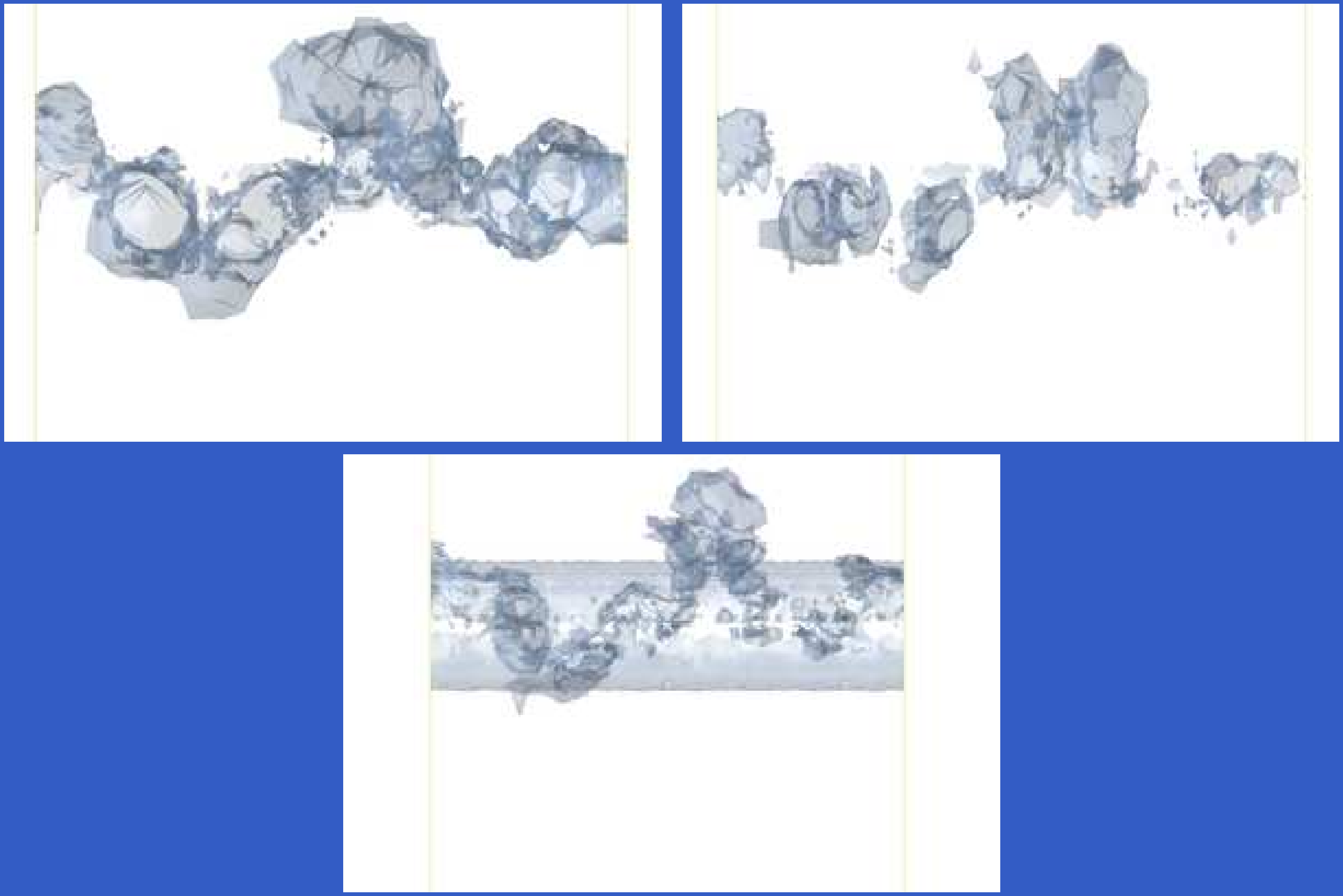
Vorticity: ω_1, ω_2, ω_3: t=1.5: $c_D = 0.28$

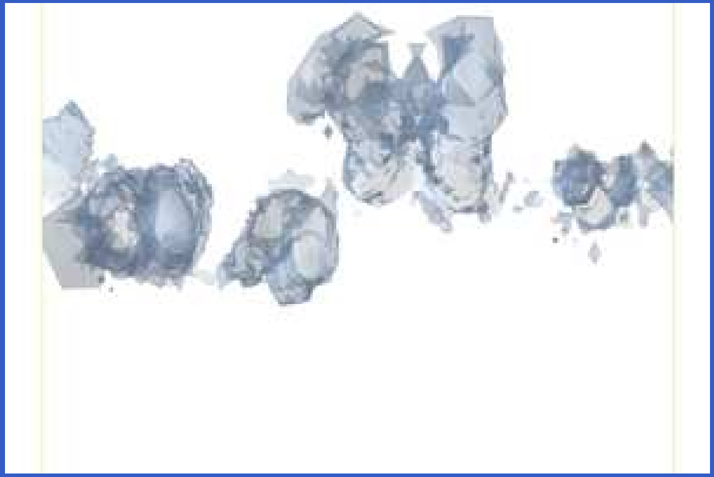

Vorticity: ω_1, ω_2, ω_3: **t=1.75:** $c_D = 0.36$



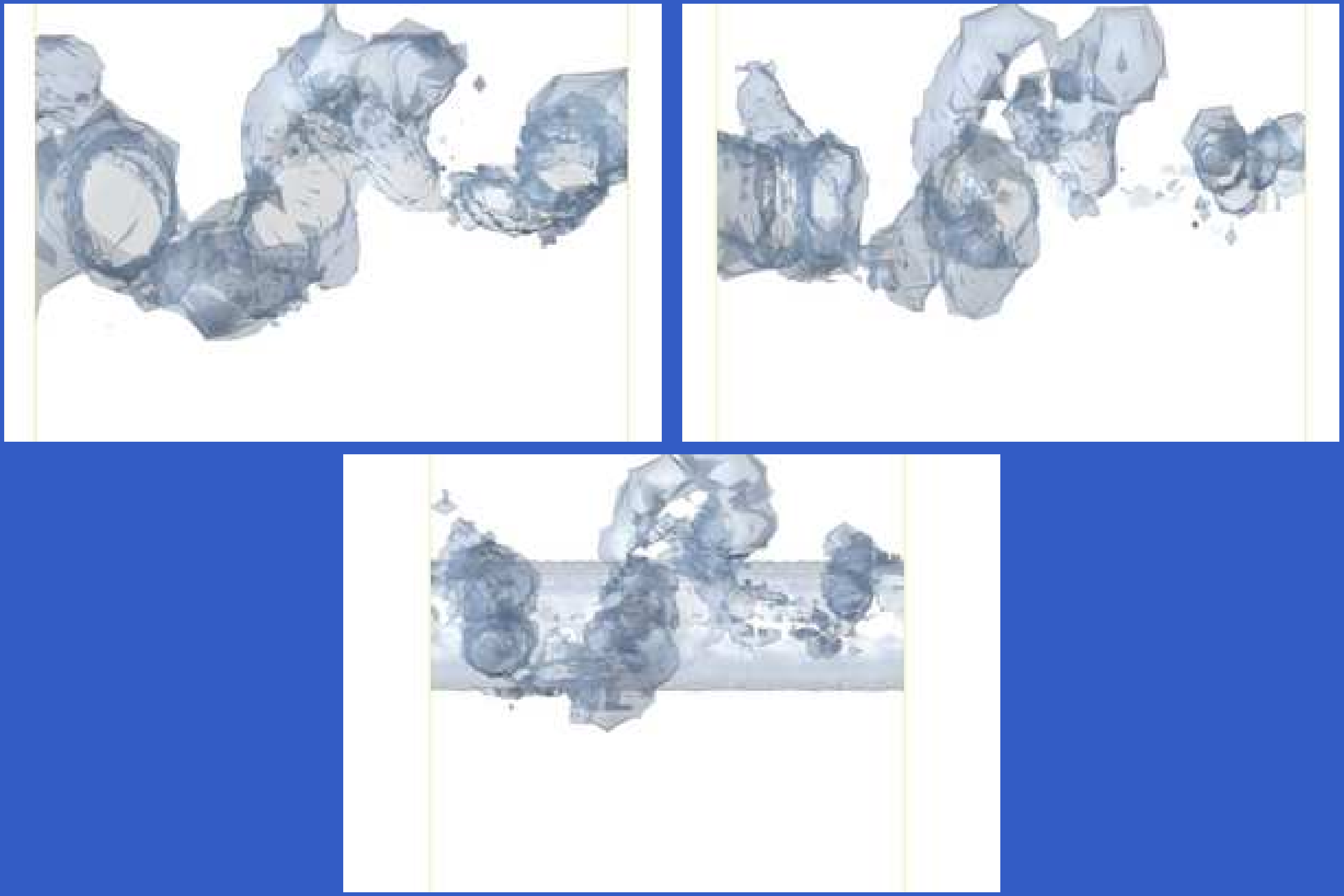
Vorticity: $\omega_1, \omega_2, \omega_3$: t=2.0: $c_D = 0.51$



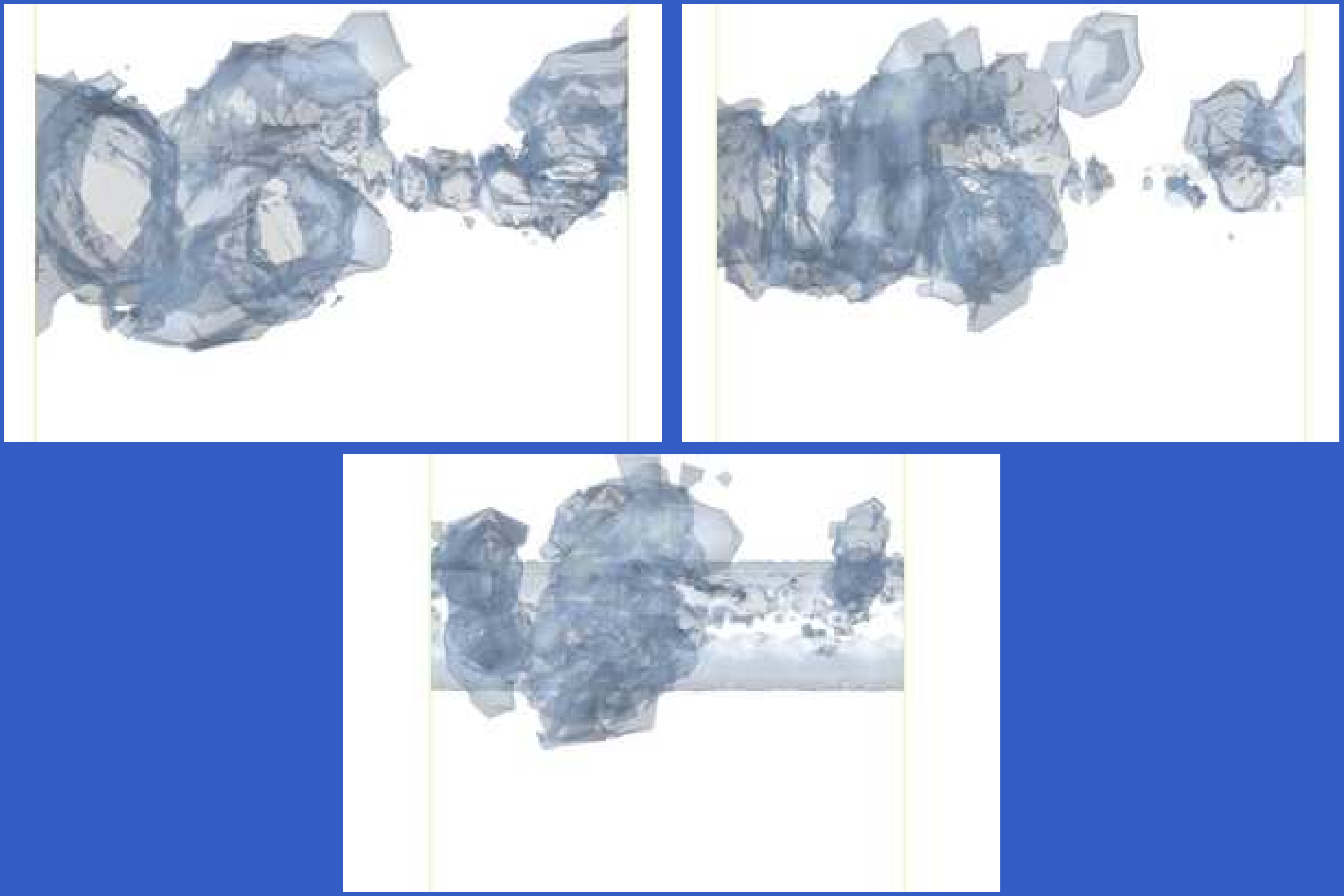
Vorticity: ω_1, ω_2, ω_3: t=2.25: $c_D = 0.78$

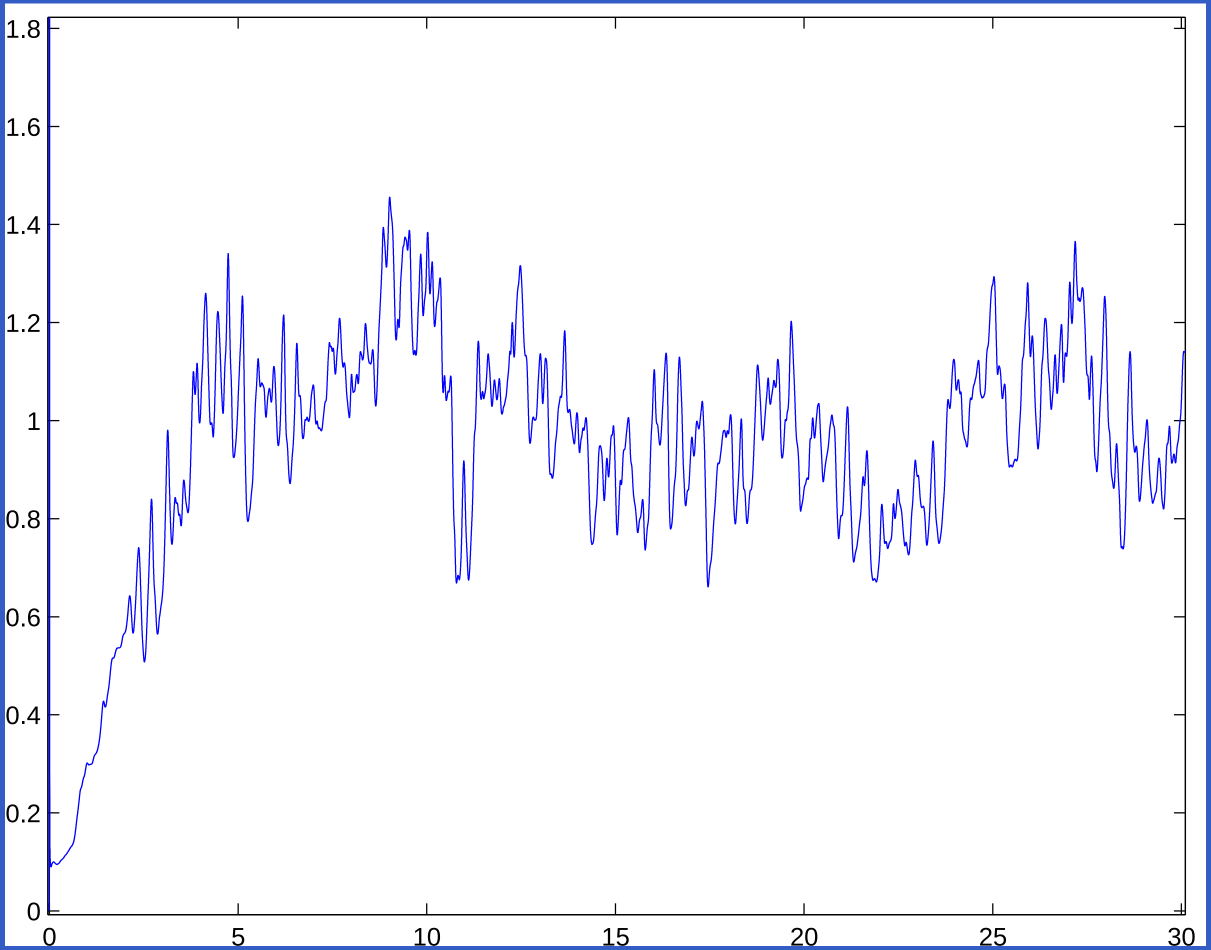

Vorticity: ω_1, ω_2, ω_3: t=2.5: $c_D = 1.14$



Vorticity: ω_1, ω_2, ω_3: t=2.75: $c_D = 1.04$



Drag Coefficient



Guadalupe Aug 20 1999

NON-EXISTENT EXACT SOLUTION

$$\dot{U} + U \cdot \nabla U + \nabla P \neq 0, \quad \nabla \cdot U \neq 0$$

- NON-EXISTENCE of (stable) POINTWISE SOL!!
- NON-EXISTENCE of (stable) EXACT WEAK SOL!!
- TURBULENCE!!
- EXISTENCE of APPROXIMATE WEAK SOL!!
- OUTPUT UNIQUENESS!! DRAG...
- TURB $=$ KIN ENER DECREASE $=$ DRAG
- IRREVERSIBILITY: Perpetuum Mobile Imposs!!

G2: Approximate Weak Solution

Find $\hat{U} = (U, P) \in V_h$ such that

$$(R(\hat{U}), \hat{v})_Q + (hR(\hat{U}), R(\hat{v}))_Q = 0 \quad \text{for all } \hat{v} \in V_h,$$

where

- V_h is a space of piecewise polynomials on a mesh in space/time with mesh size h,
- $(\cdot, \cdot)_Q$ is the =$L_2(Q)^4$-scalar product with $Q = \Omega \times I$,
- $R(\hat{U}) = (\dot{U} + (U \cdot \nabla)U + \nabla P - f, \nabla \cdot U)$ and $R(\hat{v}) = (\dot{v} + (U \cdot \nabla)v + \nabla q, \nabla \cdot v)$.

G2: Approximate Weak Solution

Find $\hat{U} = (U, P) \in V_h$ such that

$$(R(\hat{U}), \hat{v})_Q + (hR(\hat{U}), R(\hat{v}))_Q = 0 \quad \text{for all } \hat{v} \in V_h,$$

- GALERKIN: WEAK
- LEAST SQUARES weight h: STRONG
- RESIDUAL $R(\hat{U}) \approx 0$: weak-weighted strong
- SCYLLA (weak) and CARYBDIS (strong)
- Finest scale h

ENERGY ESTIMATE: TURB DISSIP

Choosing $\hat{v} = \hat{U}$:

$$\frac{1}{2}\|U(T)\|_\Omega^2 + \|\sqrt{h}R(\hat{U})\|_Q^2 = \frac{1}{2}\|U(0)\|_\Omega^2,$$

$\|\sqrt{h}R(\hat{U})\|_Q^2 \sim 1$ TURBULENT WEAK SOLUTION!!

$$\frac{1}{2}\|U(T)\|^2 << \frac{1}{2}\|U(0)\|^2,$$

- LOSS of KINETIC ENERGY: IRREVERSIB
- $\|\sqrt{h}R(\hat{U})\| \approx 1$ as $h \to 0$
- FINITE LIMIT of TURBULENT DISSIPATION

CAUSE of IRREVERSIBILITY

- SHOCKS
- TURBULENCE: $R(\hat{U})$ LARGE PW!!
- NOT STRONG SOLUTION

- APPROX WEAK SOLUTION
- WEAK UNIQUENESS

RESOLUTION of D’ALEMBERT

- Zero-Drag POT SOL UNSTABLE
- Non-Zero-Drag APPR TURBULENT SOL
- SKIN FRICTION SMALL: SLIP
- NO BOUNDARY LAYER
- ONE SEPARATION POINT
- STRONG STREAMLINE VORTICITY

PRANDTL’S SOLUTION

PRANDTL’S SOLUTION 1904

- “Motion of fluids with very little viscosity” 12 p
- BOUNDARY LAYER - VORTICITY ≈ 1
- TWO SEPARATION POINTS
- SPANWISE VORTICITY ≈ 1 by TRIPPING
- ANYTHING FROM NOTHING (very little visc)
- SMALL TURBULENT DISSIPATION
- ACCEPTED SOL in ALL FLUIDS BOOKS??
- WHAT DO YOU THINK? OK????

WIKIPEDIA

“That is, a proper understanding of the boundary layer allows us to understand how a (vanishingly) small viscosity and a (vanishingly) small viscous region can modify the global flow features. Thus, with one insight Prandtl resolved d’Alembert’s Paradox and provided fluid mechanists with the physics of both lift and form drag.”

EULER/G2

$\omega_1,\ xy$ $\omega_2,\ xy$ $\omega_3,\ xy$

$\omega_1,\ xz$ $\omega_2,\ xz$ $\omega_3,\ xz$

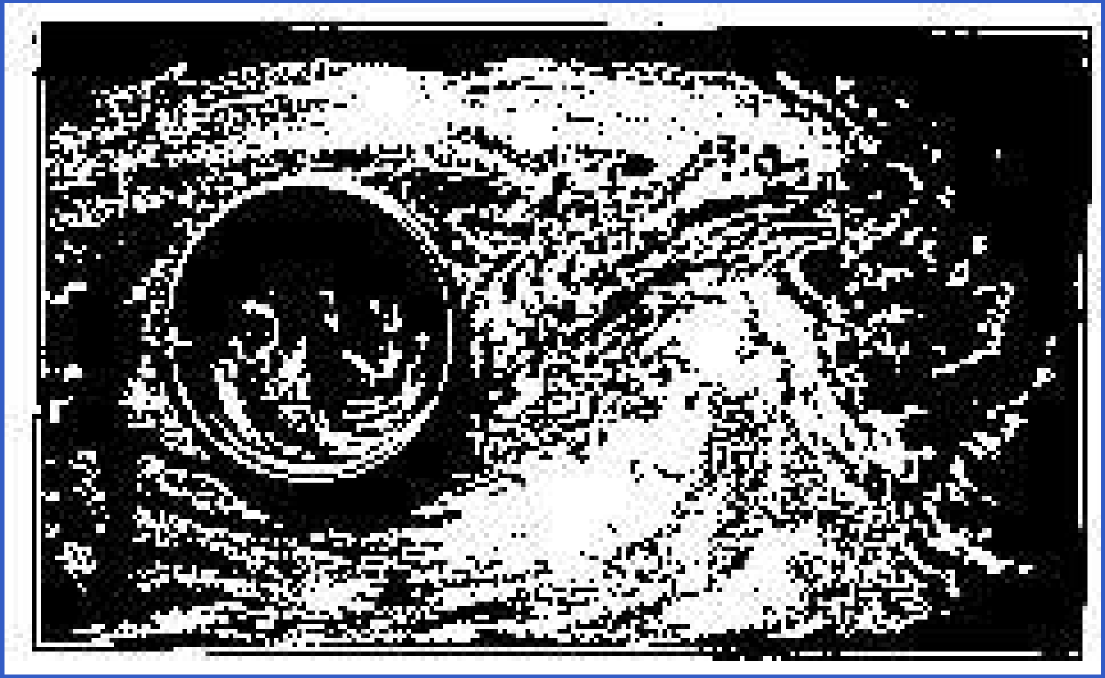
$\omega_1,\ yz$ $\omega_2,\ yz$ $\omega_3,\ yz$

$\omega_1,\ xy$ $\omega_2,\ xy$ $\omega_3,\ xy$

$\omega_1,\ xz$ $\omega_2,\ xz$ $\omega_3,\ xz$

$\omega_1,\ yz$ $\omega_2,\ yz$ $\omega_3,\ yz$

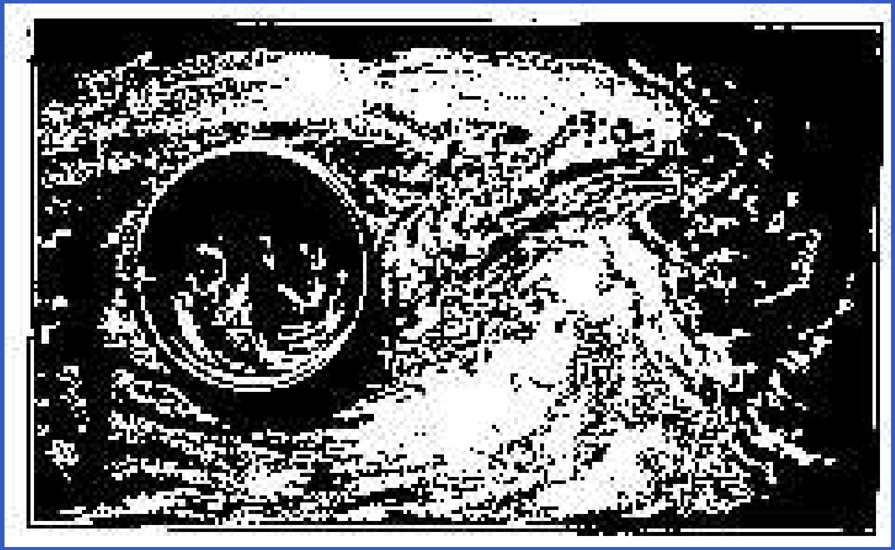
PRANDTL's EXPERIMENT

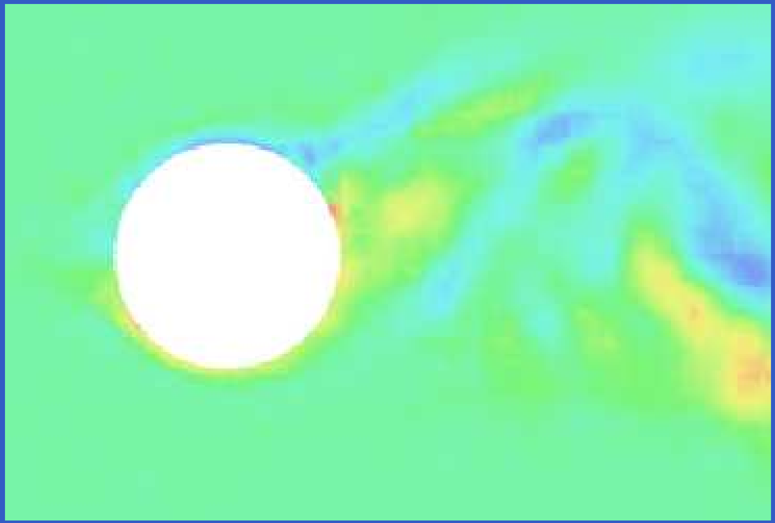


u p

u p

EXPERIMENT vs EULER/G2



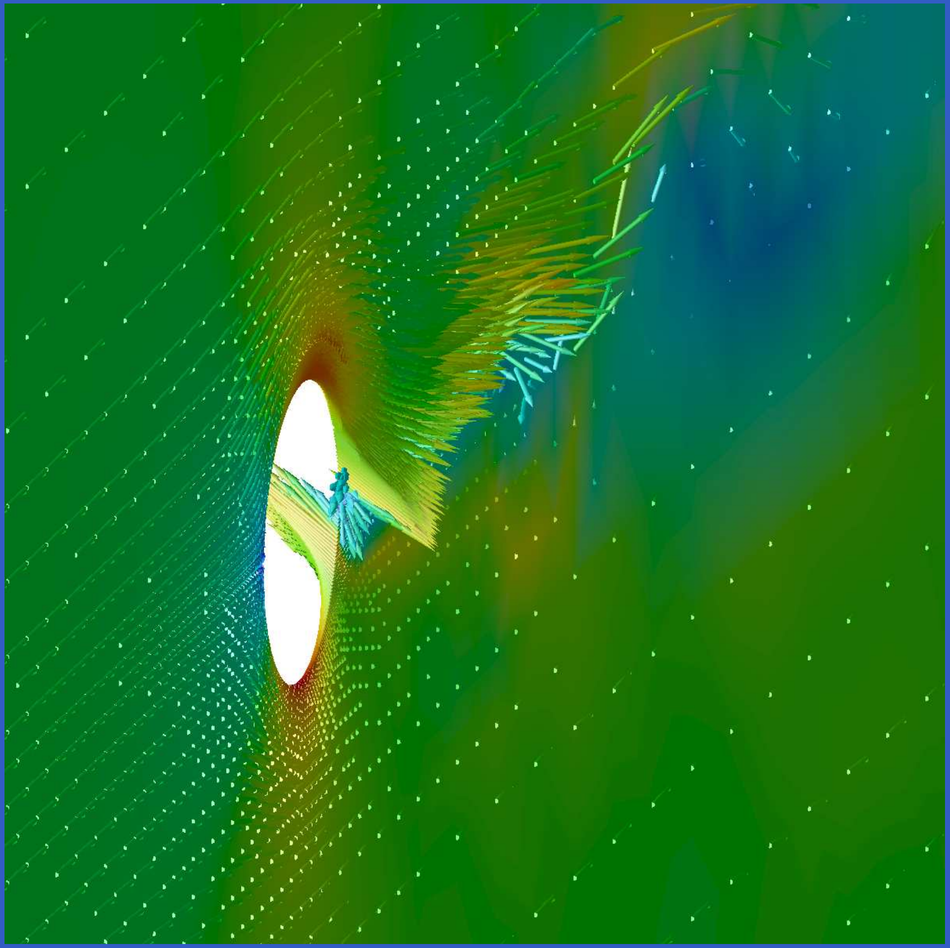


FACTS

- NO EXPERIMENTS for $Re > 10^7$
- EULER: VERY LARGE Re
- GEOPHYSICS!!

GUADALUPE

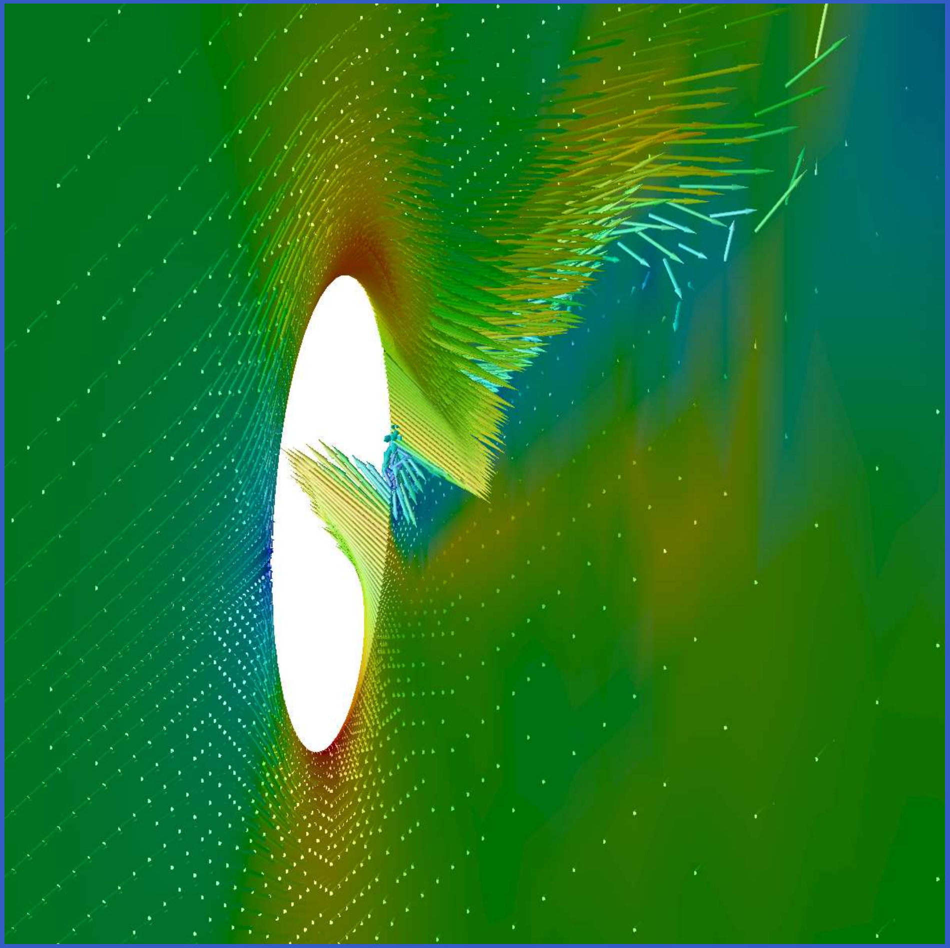
ONE SEPARATION POINT



Euler/G2:

Separation in one point

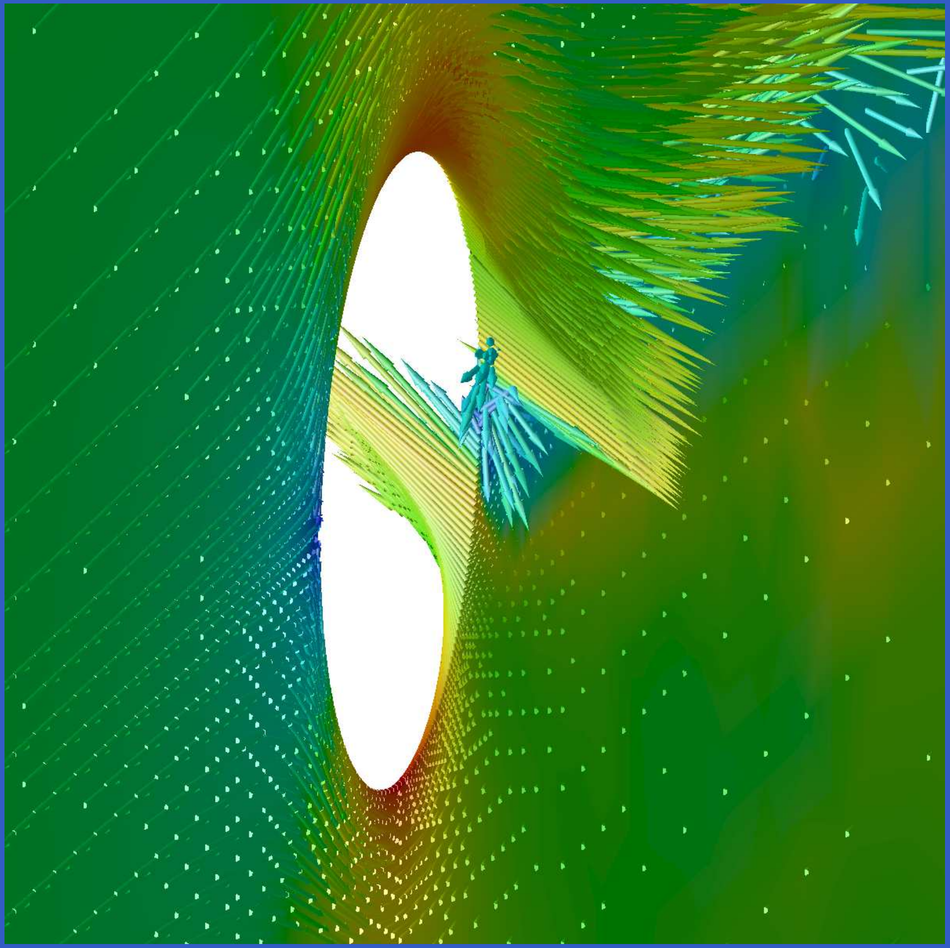
ONE SEPARATION POINT



Euler/G2:

Separation in one point

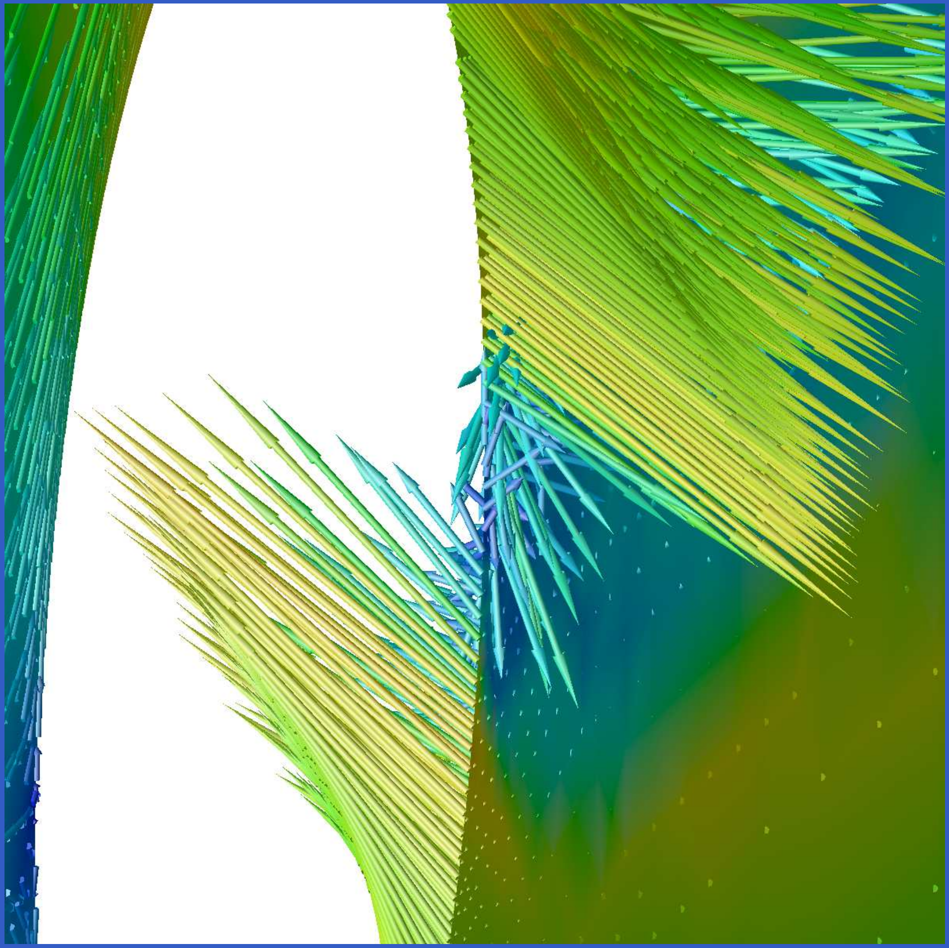
ONE SEPARATION POINT



Euler/G2:

Separation in one point

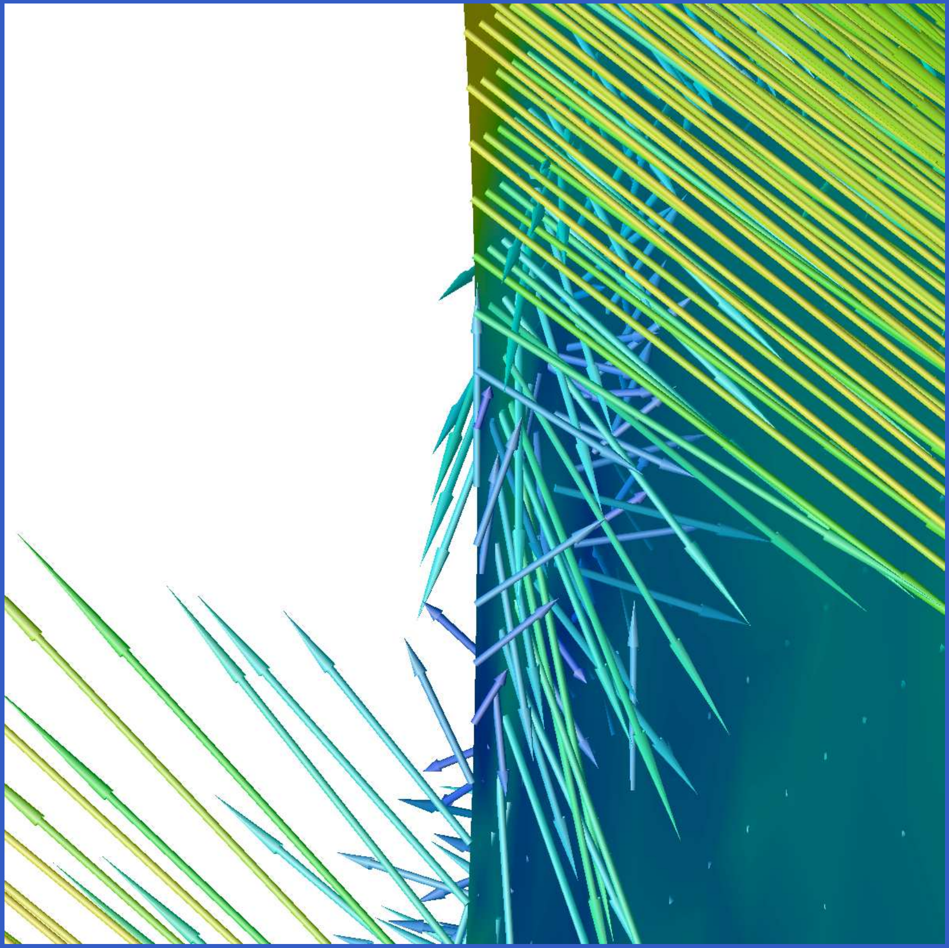
ONE SEPARATION POINT



Euler/G2:

Separation in one point

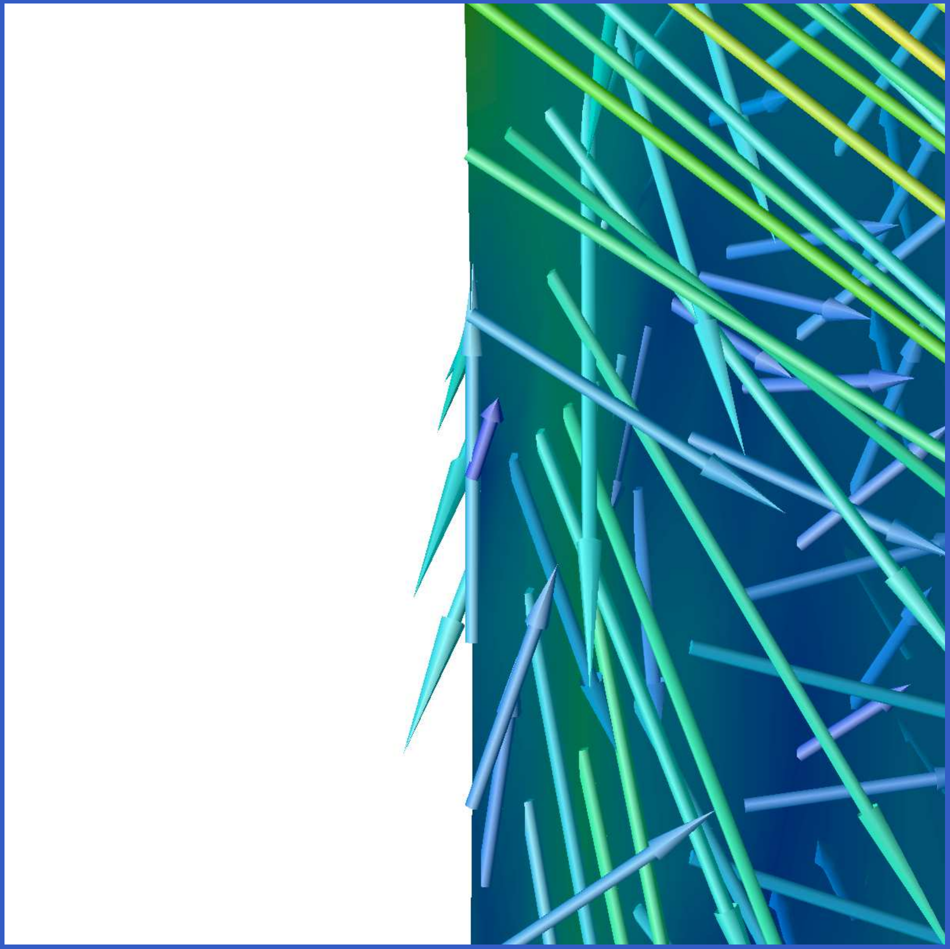
ONE SEPARATION POINT



Euler/G2:

Separation in one point

ONE SEPARATION POINT

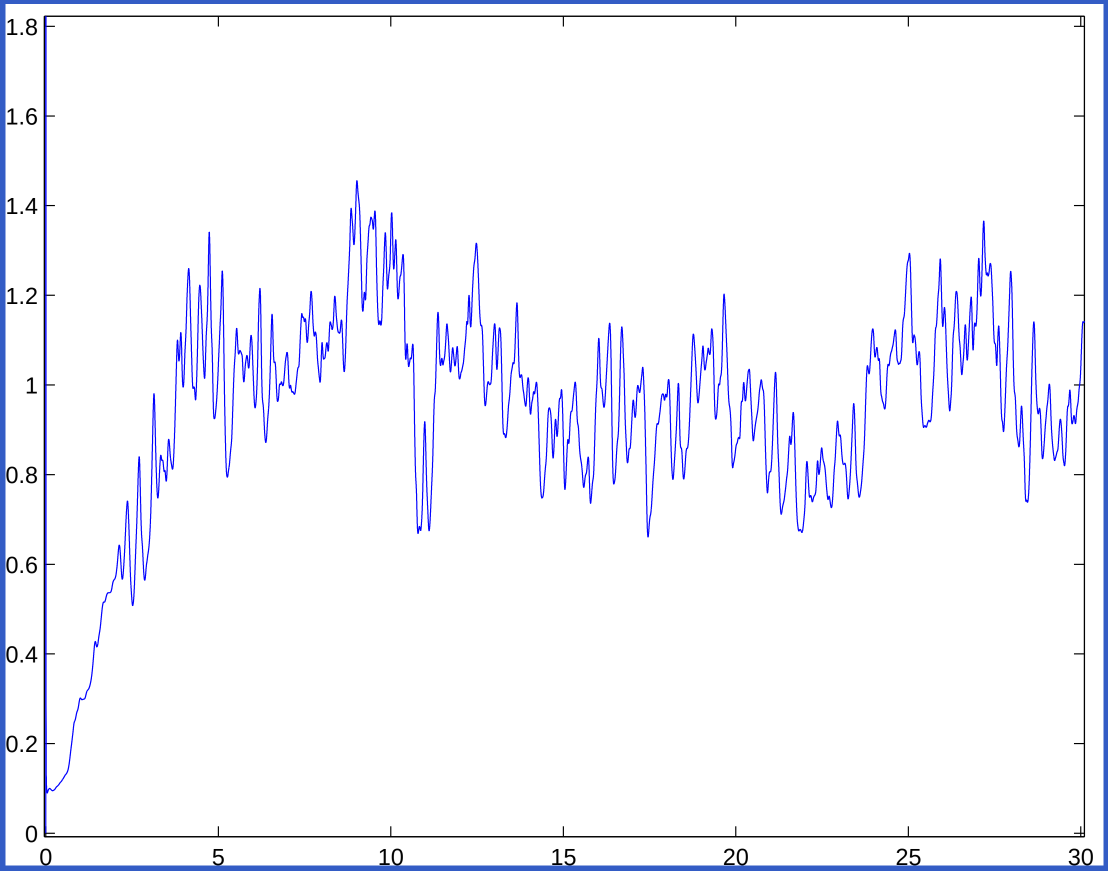


Euler/G2:

Separation in one point

DRAG EULER/G2

Euler/G2: normalized drag force as a function of time



NEW SOLUTION D'ALEMBERT

- POTENTIAL SOLUTION UNSTABLE
- LARGE REYNOLDS $=$ SMALL VISC: SLIP
- ONE SEPARATION POINT: OSCILL
- STREAMWISE VORTICITY: LOW PRESSURE
- LARGE DRAG ≈ 1
- NO SMALL CAUSE WITH LARGE EFFECT
- NEW or PRANDTL??

RESURRECTION of EULER

- EULER OK for LARGE REYNOLDS: SLIP
- TURBULENT EULER
- AVOID RESOLUT of BOUNDARY LAYER!!
- LARGE REYNOLDS COMPUTABLE on LAP-TOP!!

RESOLUTION: LOSCHMIDT

- EXACT SOL REVERSIBLE, NON-EXIST
- TURBULENT SOL IRREVERSIBLE

- STATIST MECH: NON-SCIENTIFIC:
- MICROSCOPIC GAMES of ROULETTE??
- NEW or STATIST MECH??

RESOLUTION: SOMMERFELD

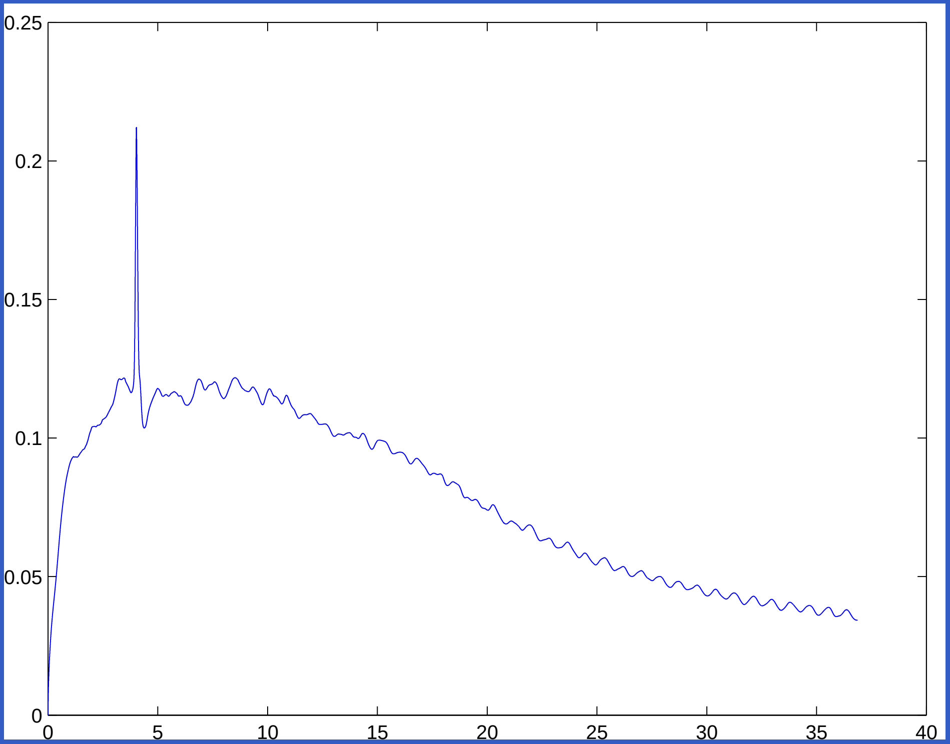
- COUETTE UNSTABLE POT SOL
- LINEAR PROBLEM NON-NORMAL:
- EIGENVALUE ANAL WRONG
- SCHLICHTING: COUETTE TOO SIMPLE??
- NEW or SCHLICHTING??

MORE MYSTERIES

- GIBBS 1875: JOULE-THOMSON
- BLACK-BODY: Planck 1900
- QUANTUM MECHANICS

JOULE-THOMSON EXP 1845

MOMENTUM



BLACK-BODY

- ABSORBS ALL FREQ
- RADIATES LOW FREQ
- IRREVERSIBLE
- COMPUTATIONAL CUT-OFF
- ANALOG CUT-OFF??

QUANTUM MECHANICS

- STATIST INTERPRET (Copenhagen)
- EINSTEIN + SCHRÖDINGER PROTEST
- COMPUTATIONAL INTERPRETATION??
- EACH ELECTRON COMPUTES ITS STATE
- LEIBNIZ MONAD THEORY

FINITE PREC + STABILITY

- FINITE PREC COMPUTATION
- ANALOG or DIGITAL
- STABILITY
- POINTWISE INDETERMINATE
- MEAN-VALUE DETERMINATE
- NO STATISTICS!!

NEW FOUND of THERMODYNAMIC

- 1st LAW: ENERGY CONSERVATION
- 2nd LAW: ENTROPY-TURBULENT DISSIP
- FINITE PRECISION G2 COMPUTATION
- 2nd LAW from 1st LAW $+$ G2
- NO STATISTICS

HAMILTONIAN PHYSICS

- INVISCID: EULER
- PARTICLE MECHANICS
- QUANTUM MECHANICS
  - EXACT SOL NON-EXISTENT: REVERSIBLE
  - APPROX SOL EXISTENT: IRREVERSIBLE
- ARROW of TIME

New Foundation: Euler/G2

- WORLD: CLOCK with FINITE PREC
- RESURRECTION of DETERMINISM
- NO STATISTICS
- EULER/G2 $\equiv$ PHYSICS ??

A Posteriori Error Estimate

OUTPUT UNIQUENESS:

$$|c_D(\hat{U}) - c_D(\hat{W})| \leq S(\|hR(\hat{U})\|_Q + \|hR(\hat{W})\|_Q)$$

where

- $c_D(U)$ drag coeff. for G2 solution U mesh h.
- S Stability factor depending on derivatives of dual solution
- $\|hR(\hat{U})\|_Q \sim \sqrt{h}$ small
- $\|R(\hat{U})\|_Q \sim 1/\sqrt{h}$ large

Stability Factor S

Linearized Dual Problem: $\hat{\phi} = (\phi, \theta)$

$$-\dot{\phi} - U \cdot \nabla\phi + \nabla U \cdot \phi + \theta = \psi, \quad \nabla \cdot \phi = 0, \quad \phi(T) = 0$$

Convection-reaction problem

$$\psi(t) = 1/\tau \quad \text{for } T - \tau < t < T$$

$$S = \|D\hat{\phi}\| / \|\psi\|$$

- ψ depends on Output (drag lift ...)
- React coeff ∇U oscillating in turbulent flow
- Cancellation

Error Representation

$$|c_D(\hat{U})-c_D(\hat{W}) = \int \Big((R(\hat{U}),\hat{\phi}-\hat{\phi}_h)+(hR(\hat{U}),R(\hat{\phi}_h))\ldots..\Big)dxdt$$

$$|c_D(\hat{U}) - c_D(\hat{W})| \leq S(\|hR(\hat{U})\|_Q + \|hR(\hat{W})\|_Q)$$

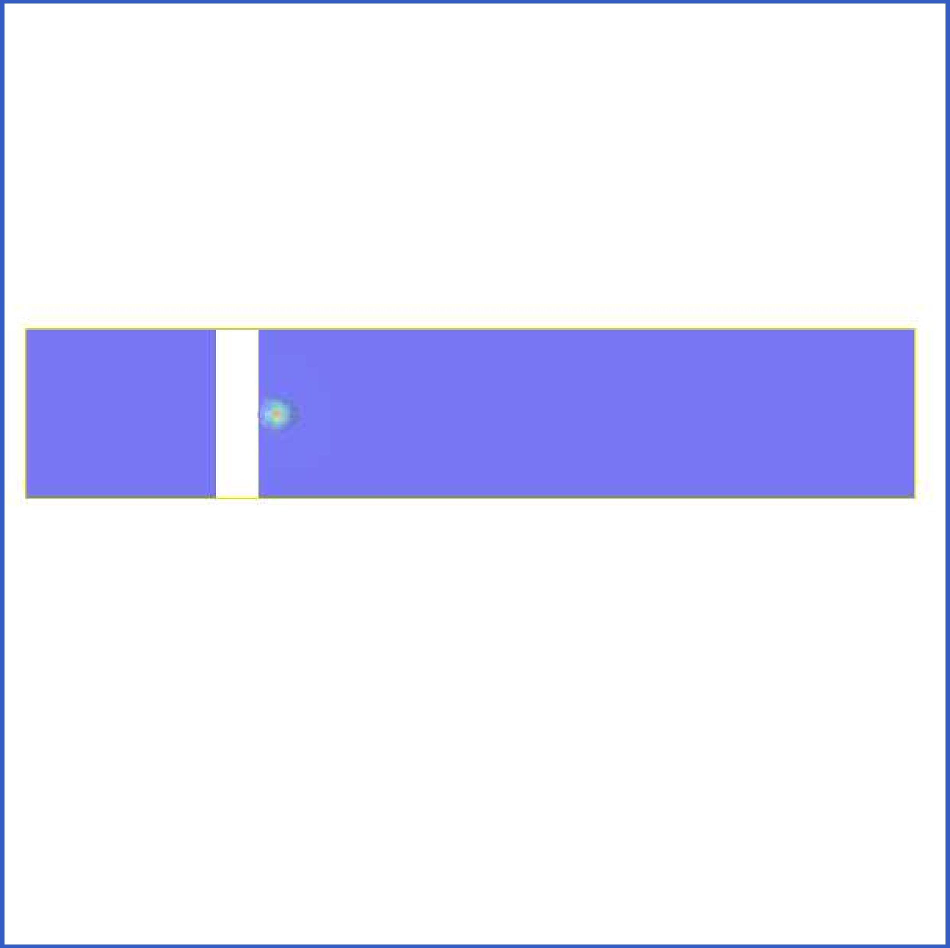
Mixing: Laminar to Turbulent

- Reaction coefficient ∇U oscillating for $T - \tau < t < T$
- CANCELLATION: S NOT LARGE
- Stable output
- Output uniqueness

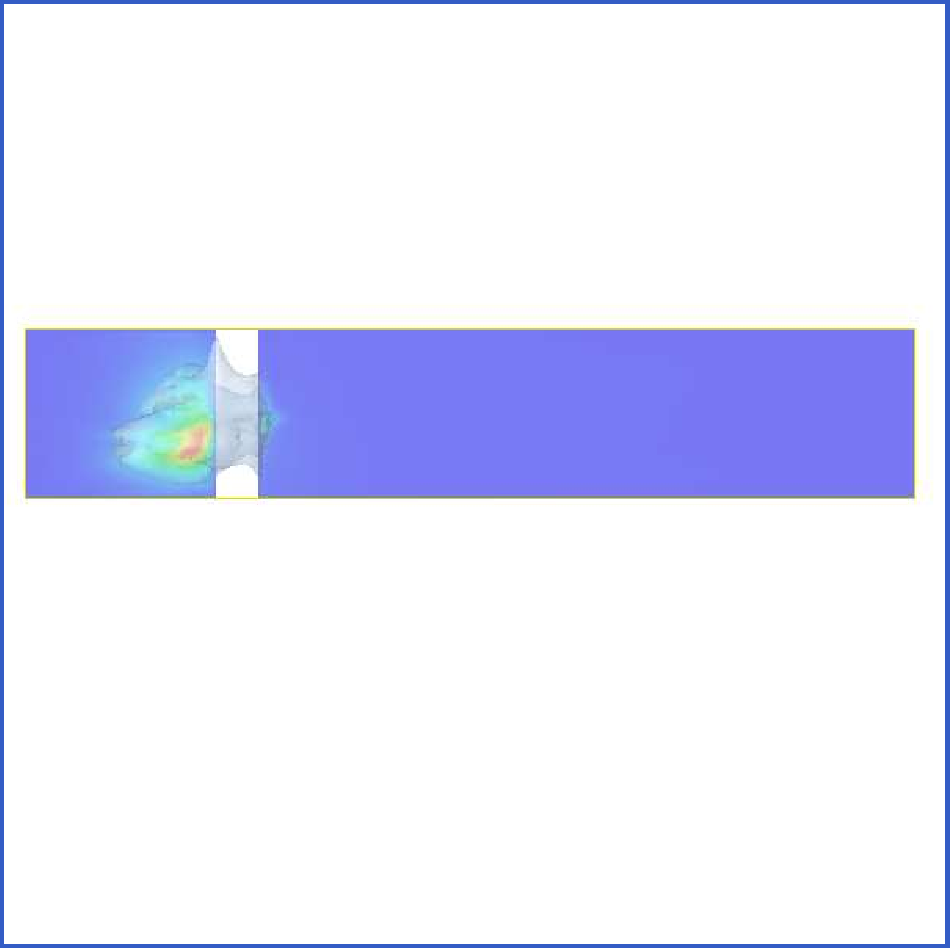
Unmixing: Turbulent to Laminar

- Reaction coefficient ∇U not oscillating for $T - \tau < t < T$
- NO CANCELLATION: S LARGE:
- Unstable output
- Output non-uniqueness
- Strong sensitivity to regular perturbations

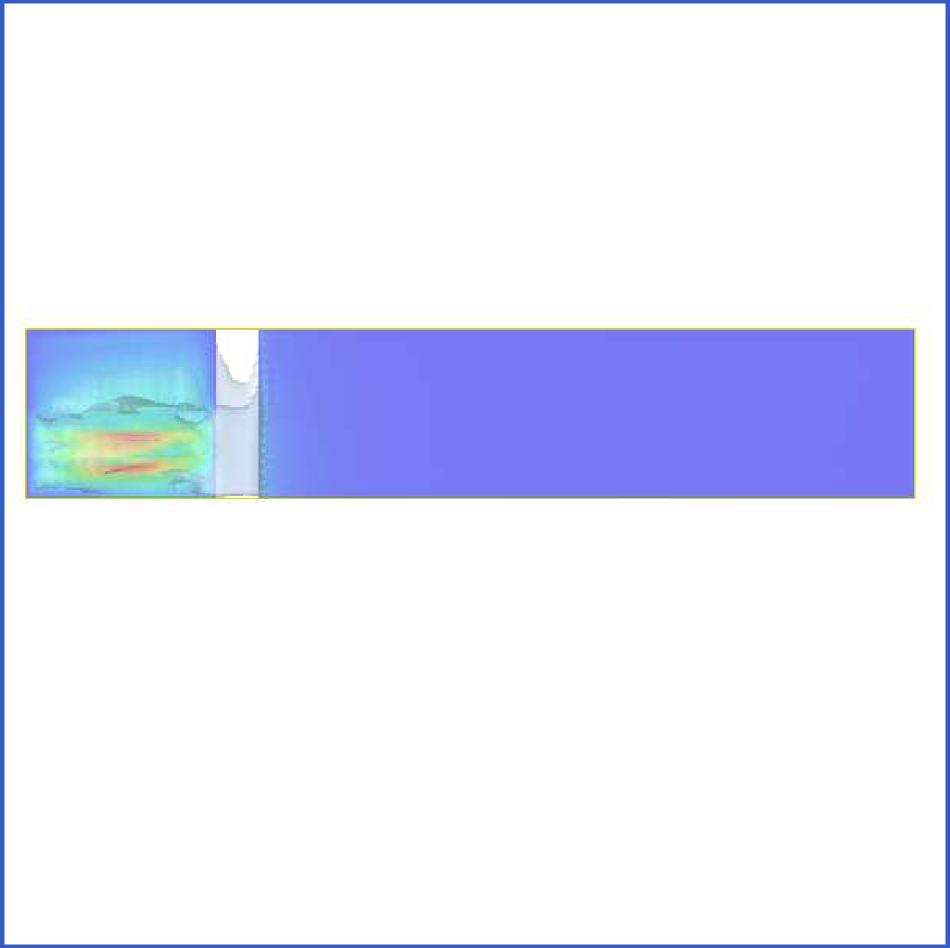
Dual d'Alembert xy 0



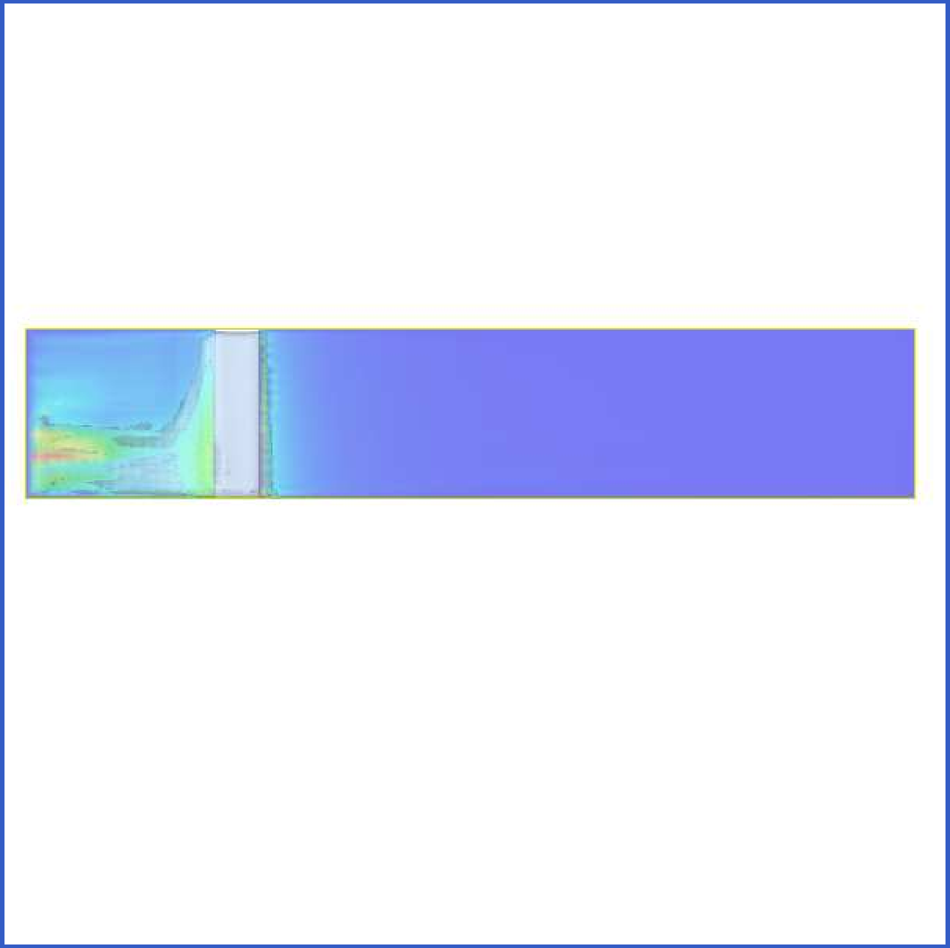
Dual d’Alembert xy 0.25



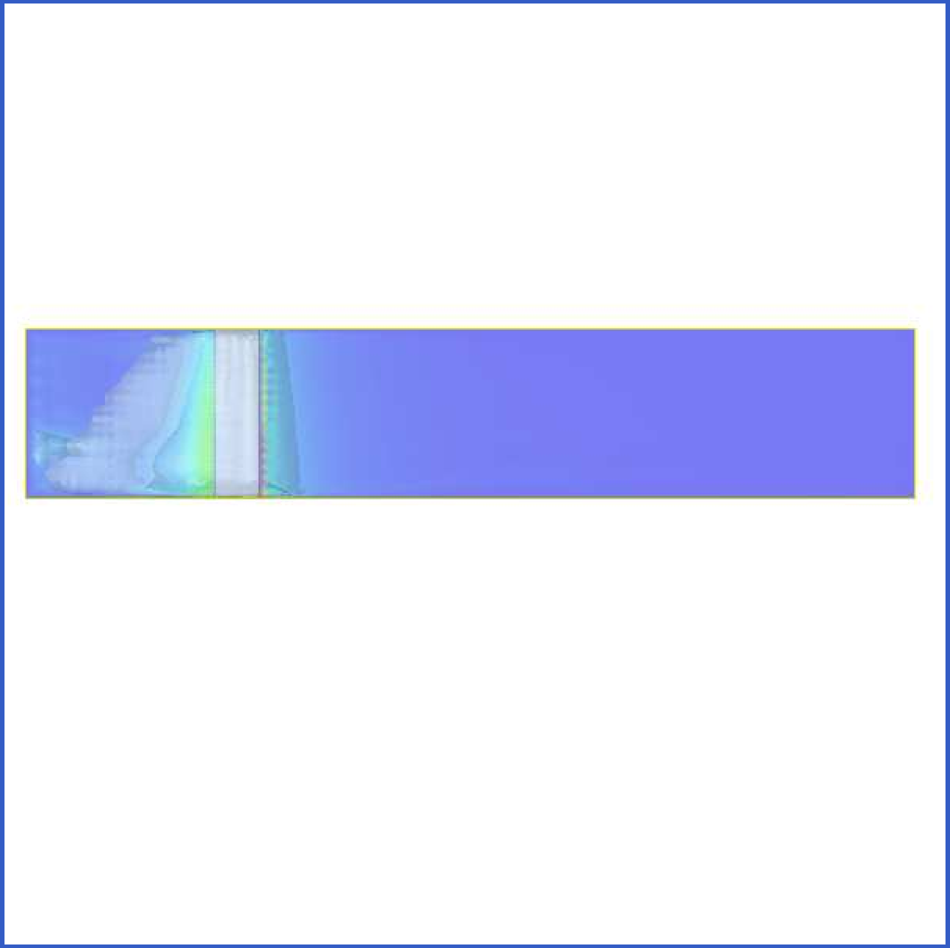
Dual d'Alembert xy 0.5



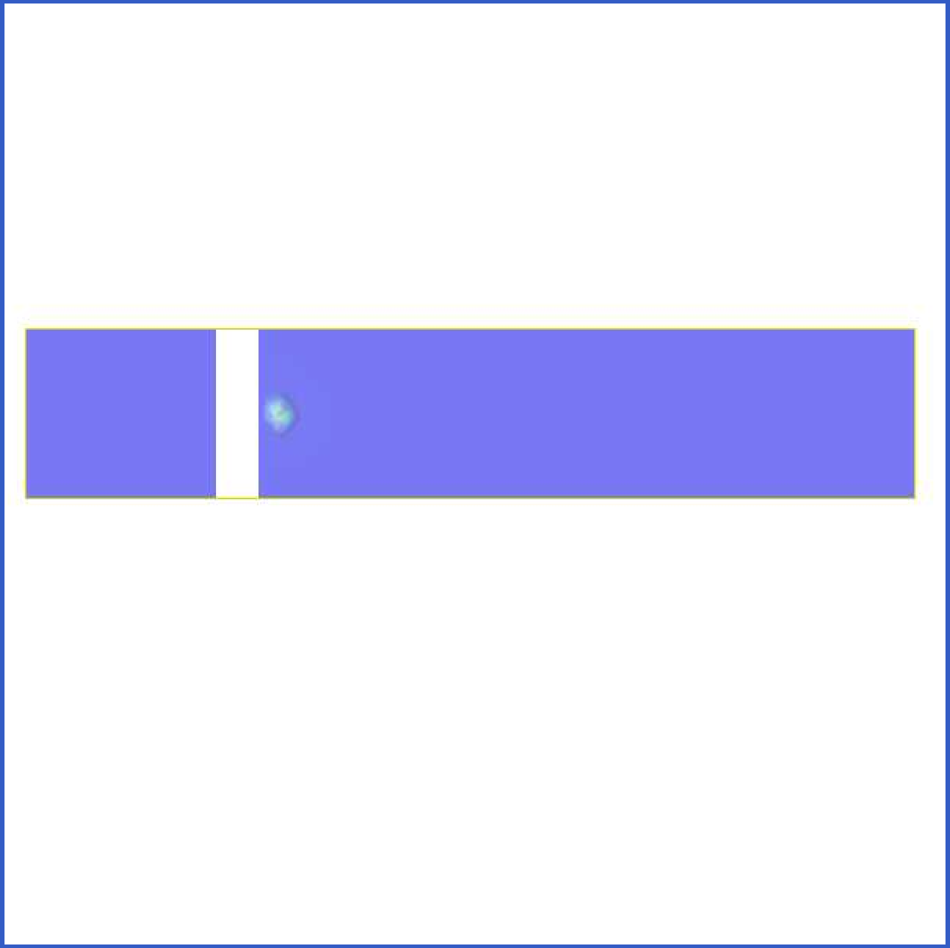
Dual d'Alembert xy 0.75



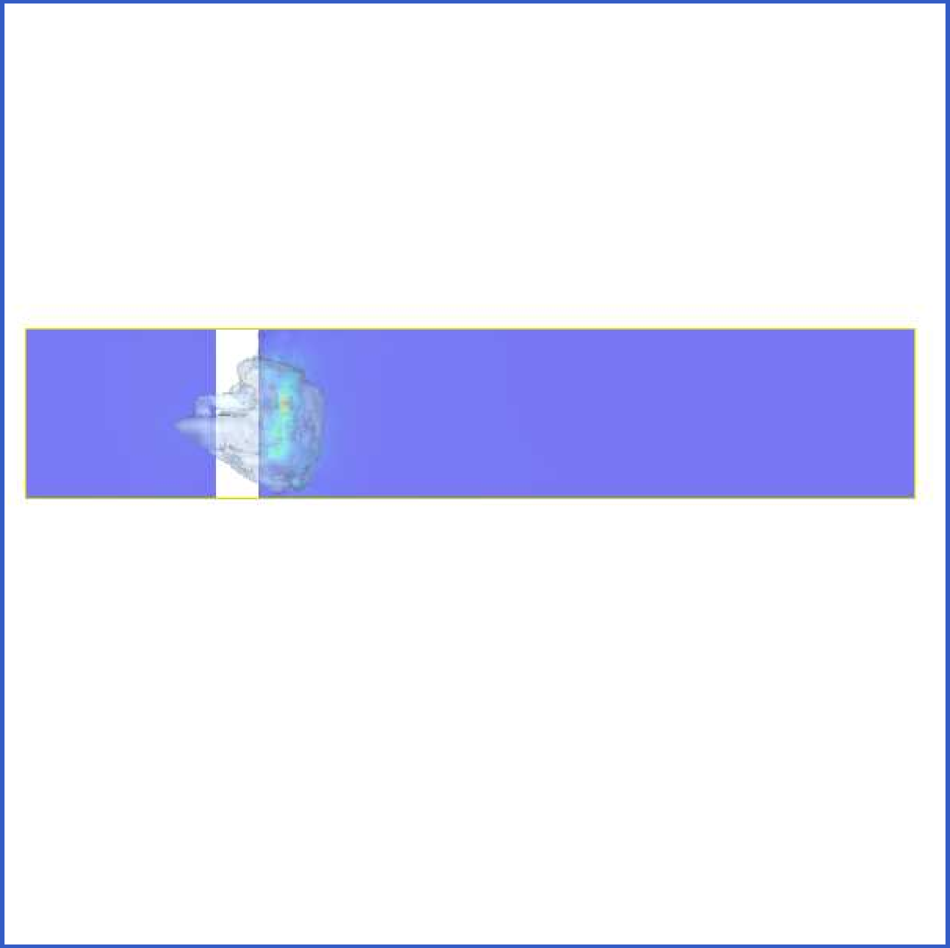
Dual d'Alembert xy 1



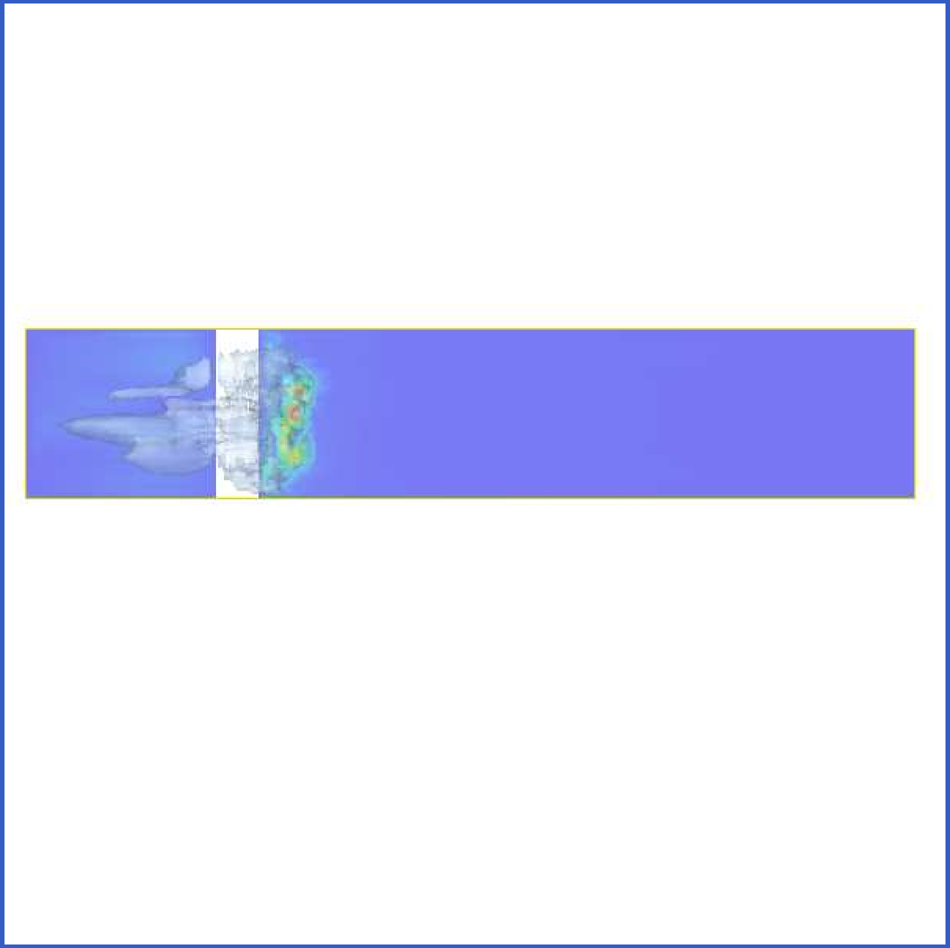
Dual Turb xy 0



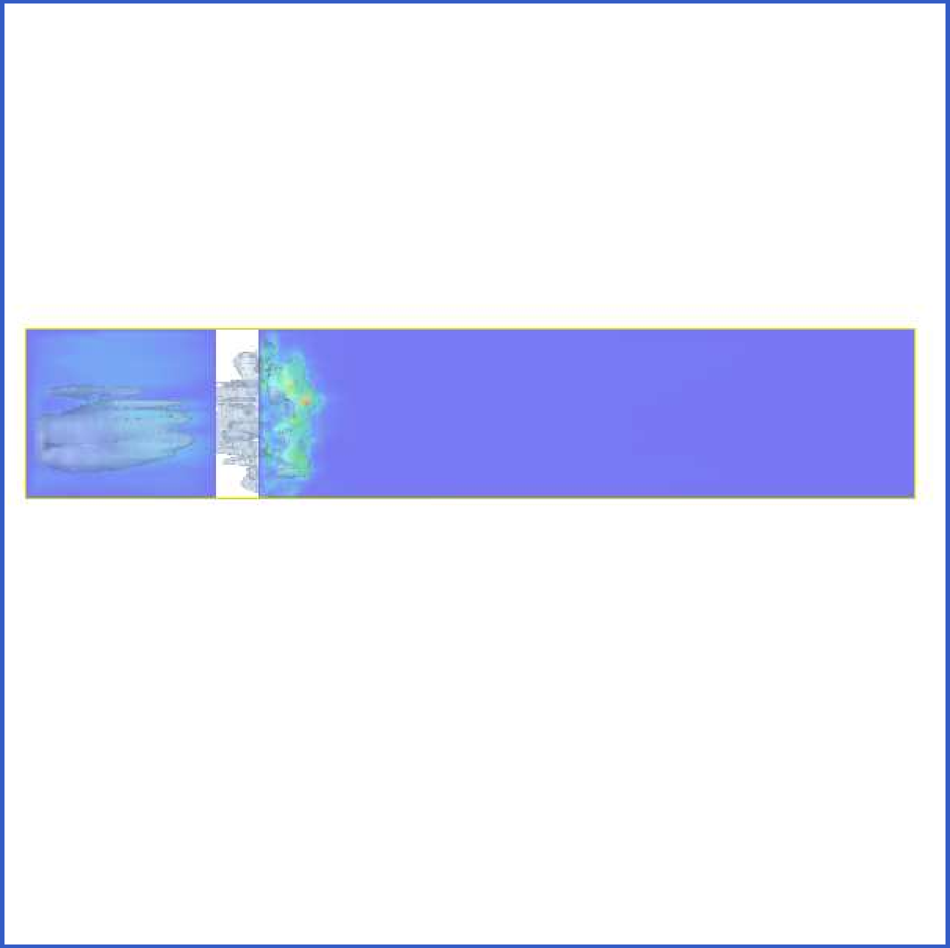
Dual Turb xy 0.25



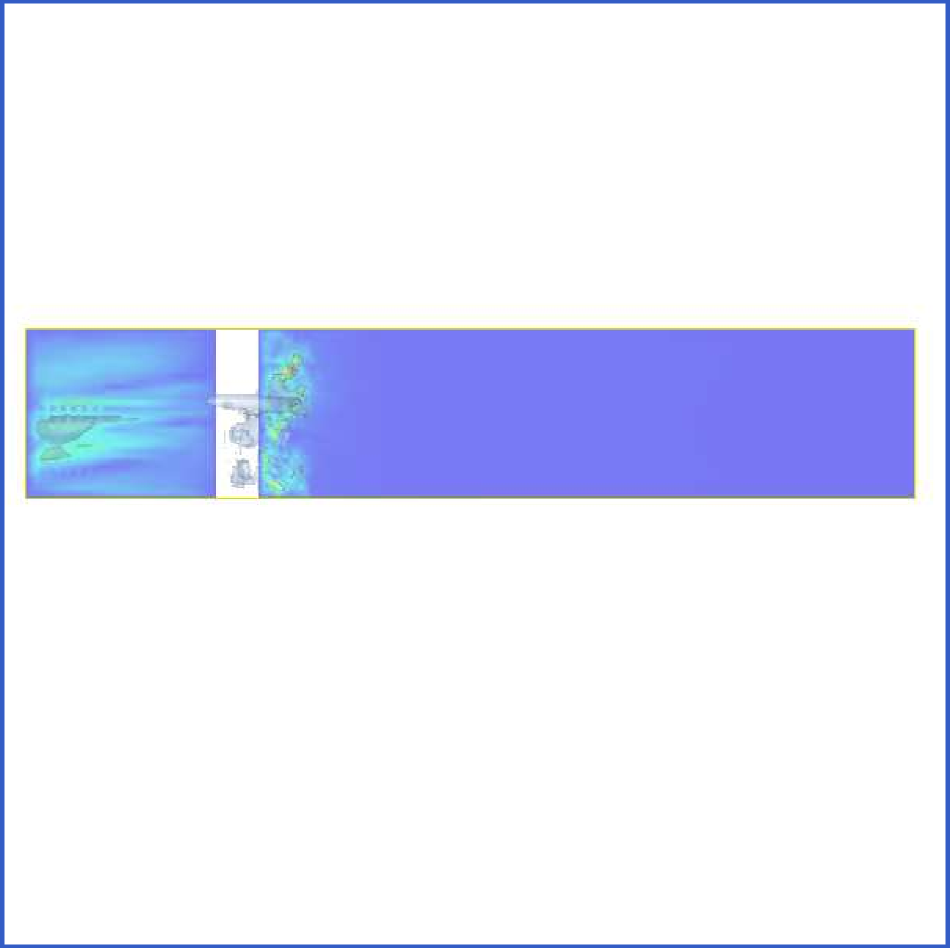
Dual Turb xy 0.5



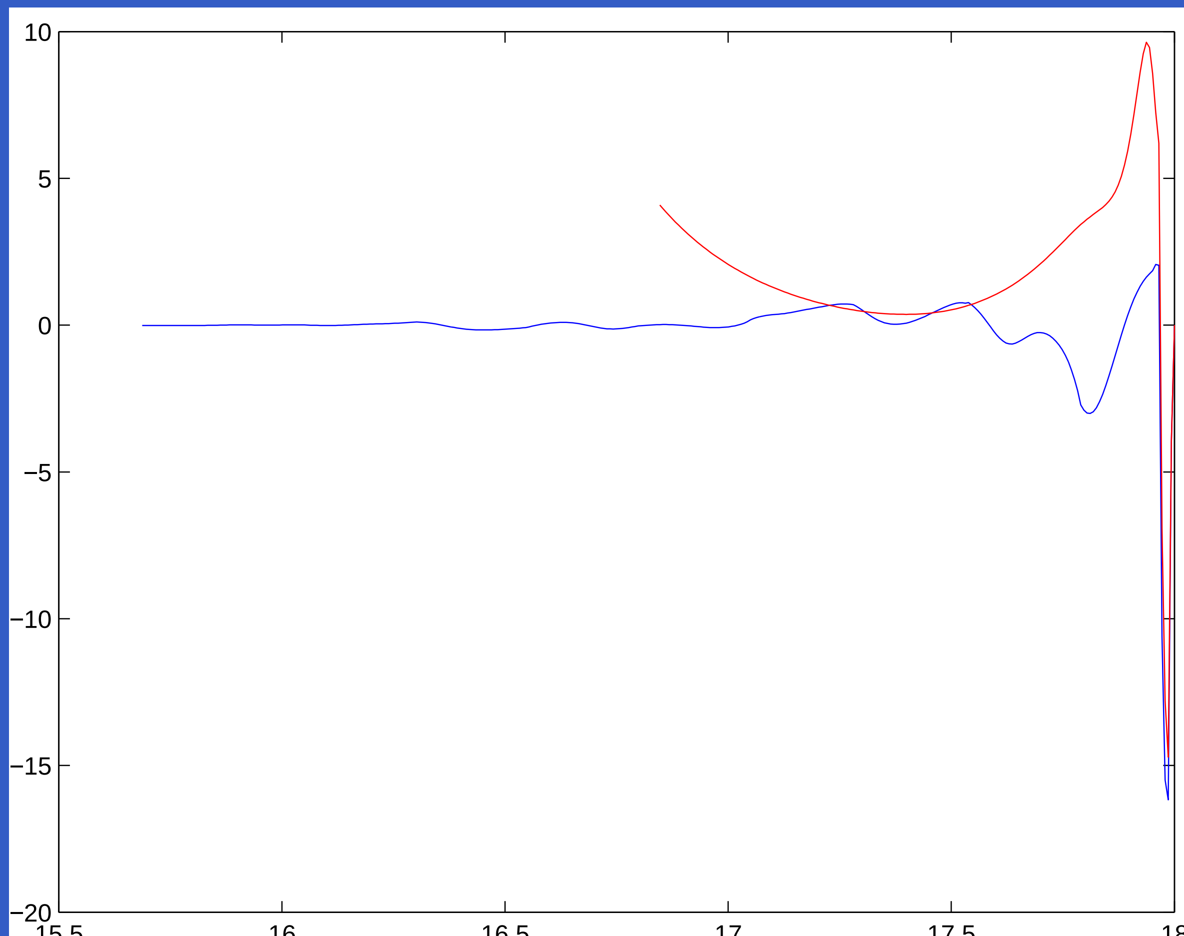
Dual Turb xy 0.75



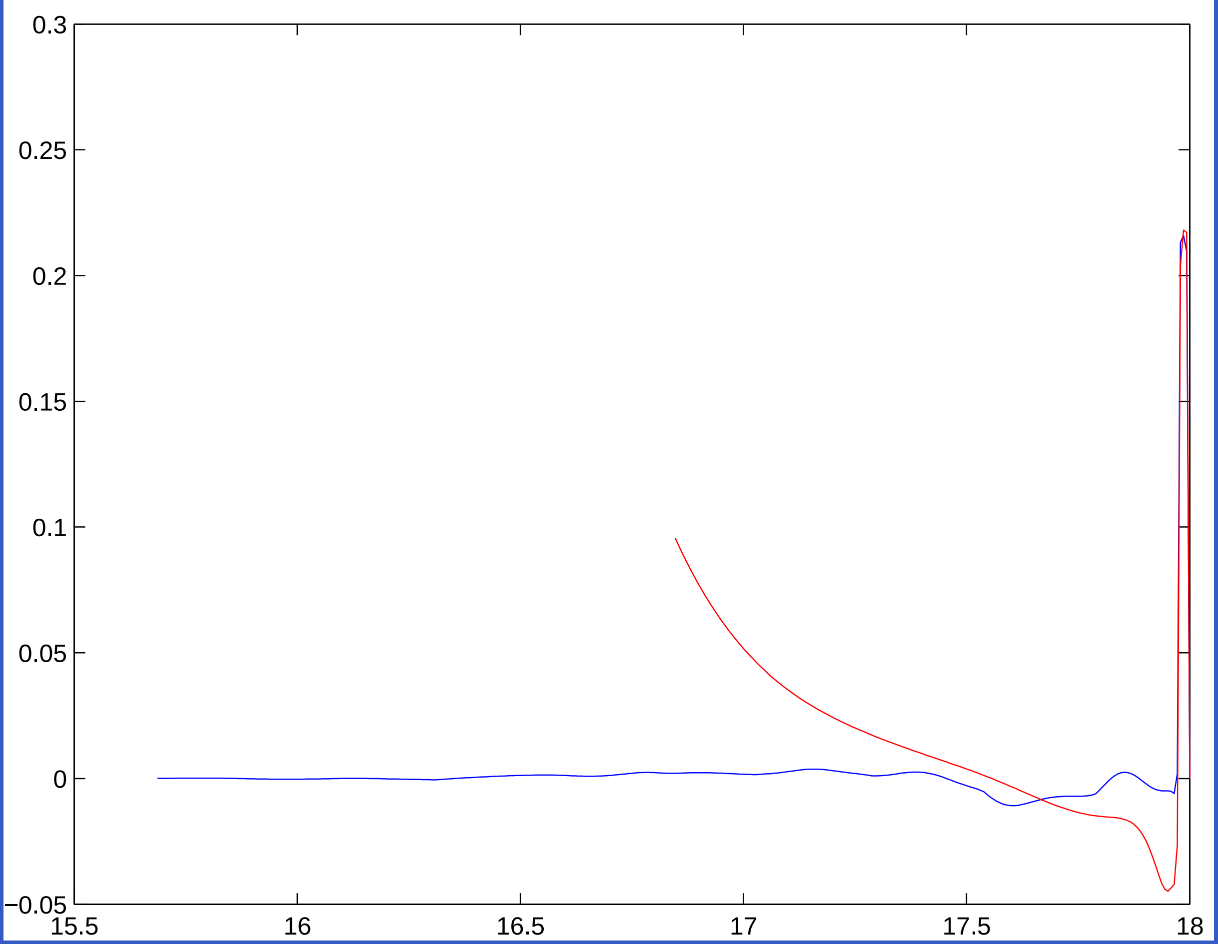
Dual Turb xy 1



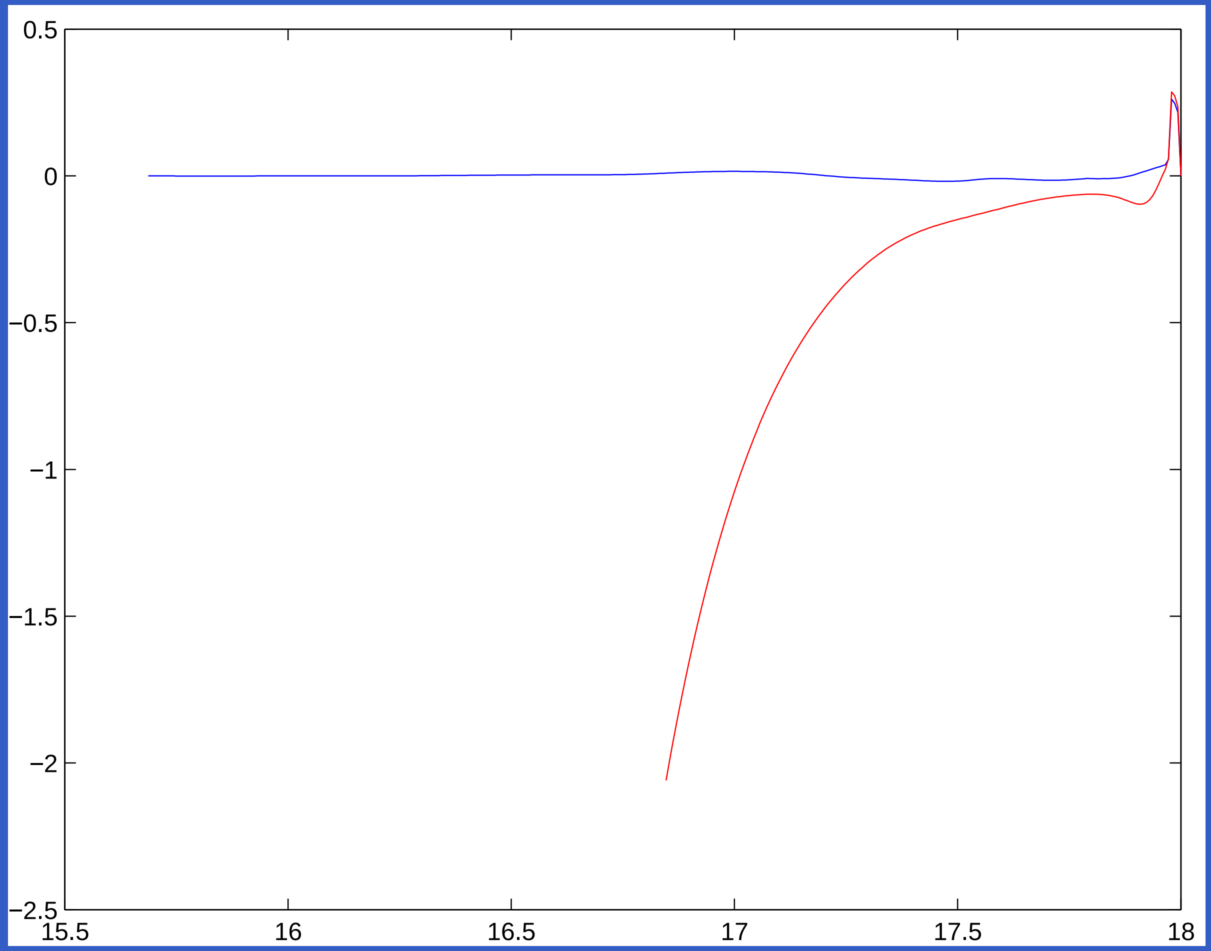
Dual Turb/dAl Integral p



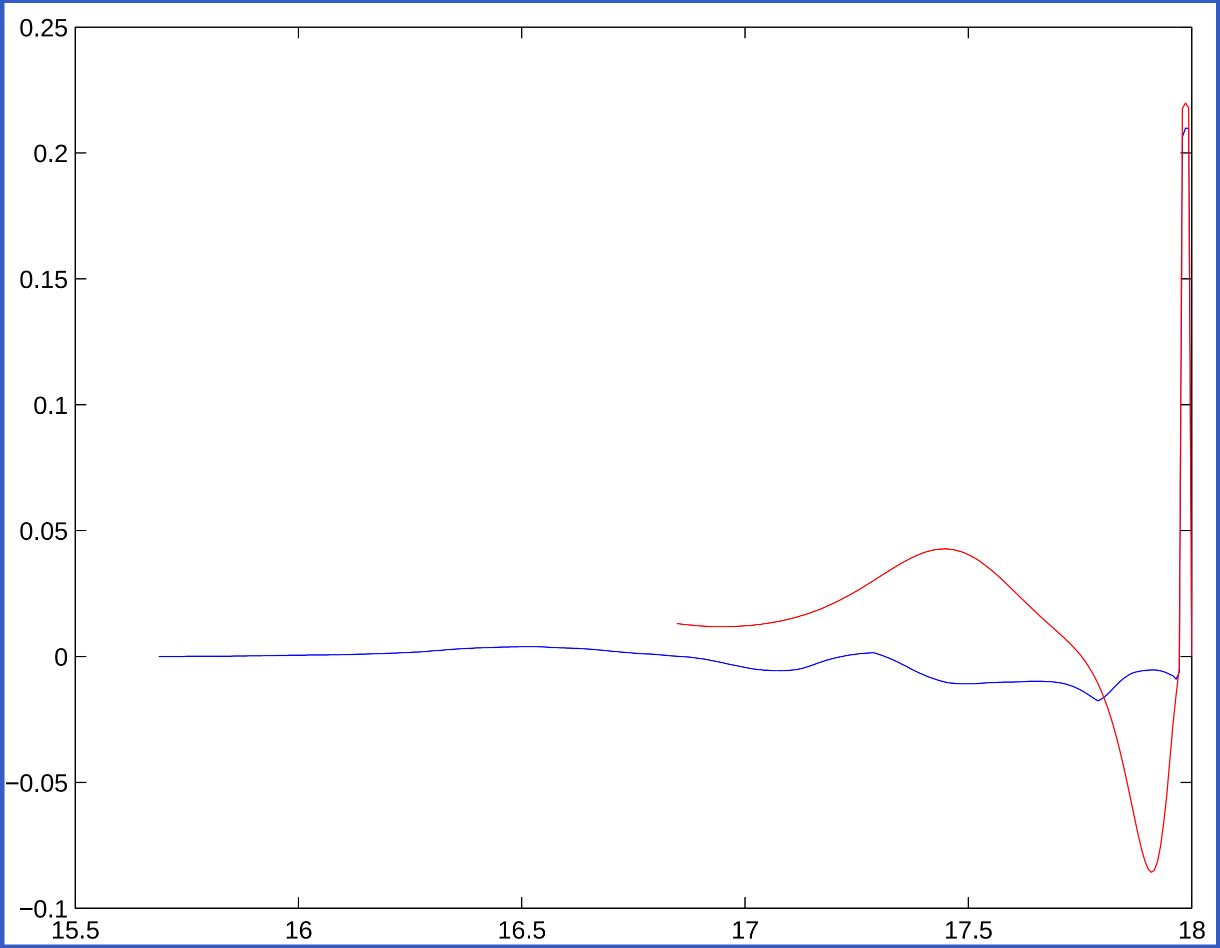
Dual Turb/dAl Integral u1



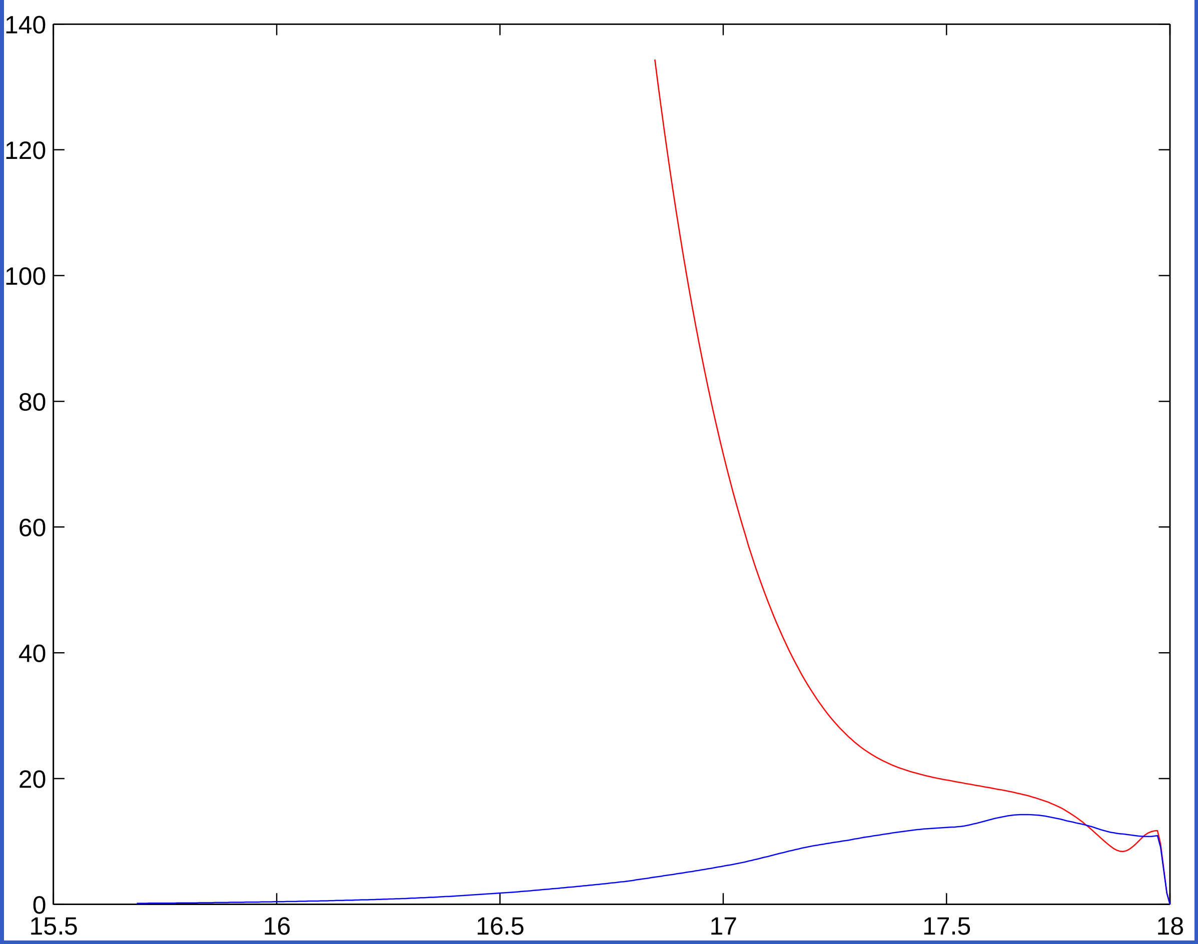
Dual Turb/dAl Integral u2



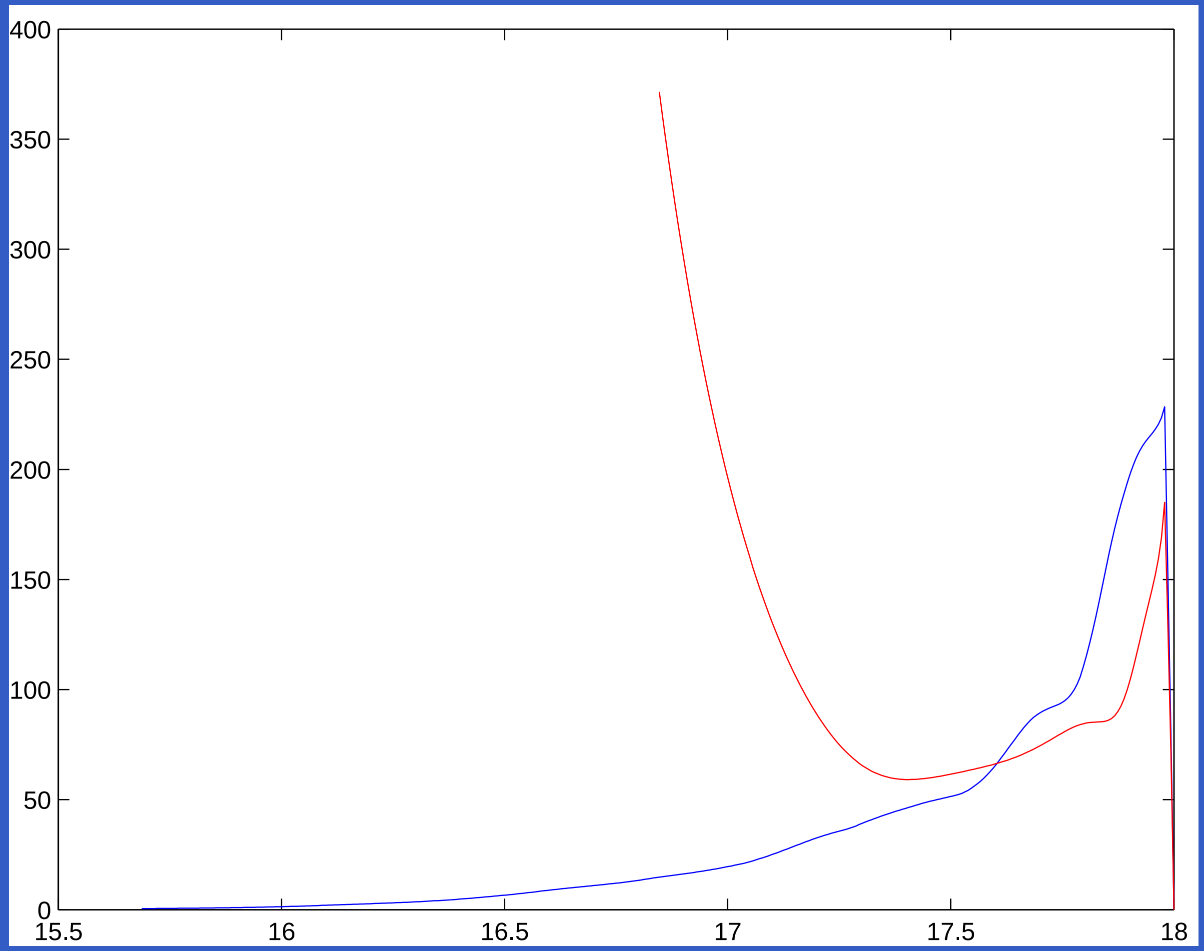
Dual Turb Integral u3



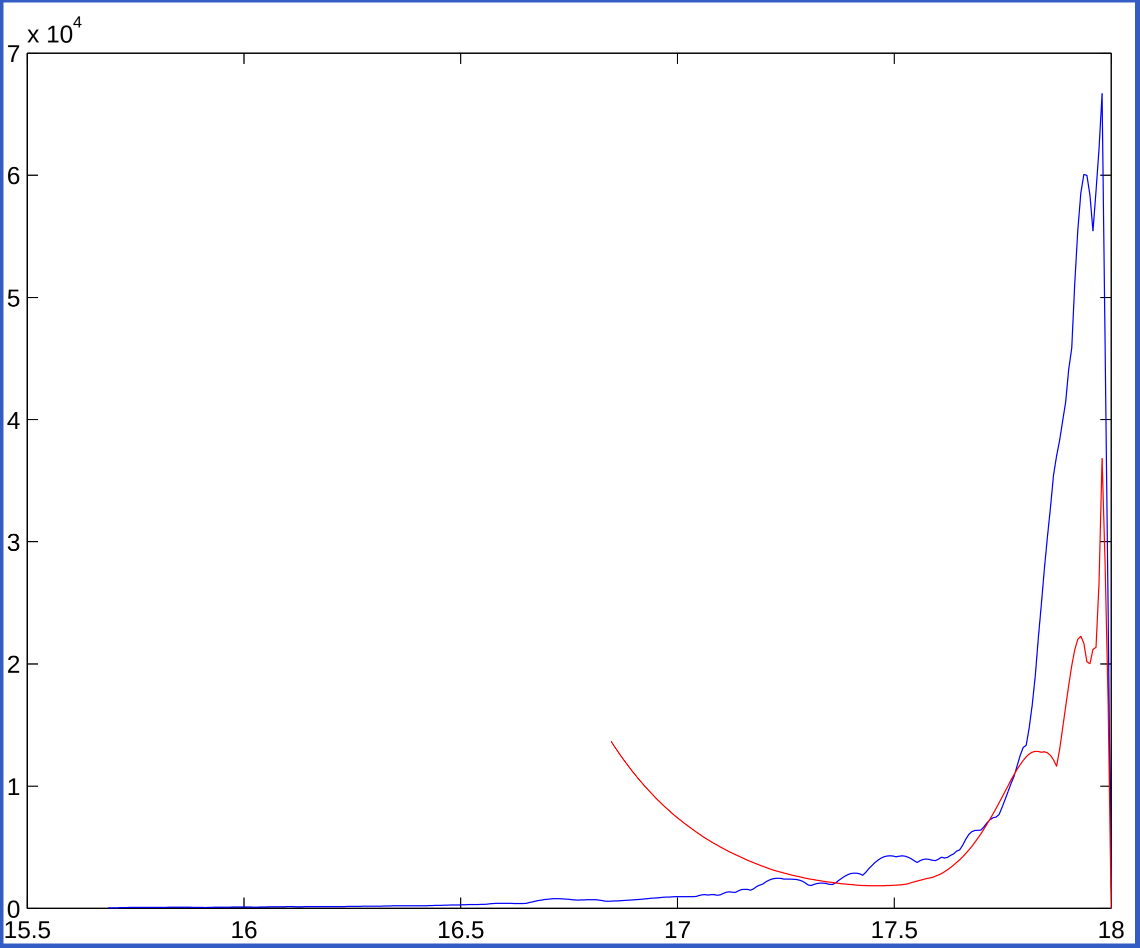
Dual Turb L1



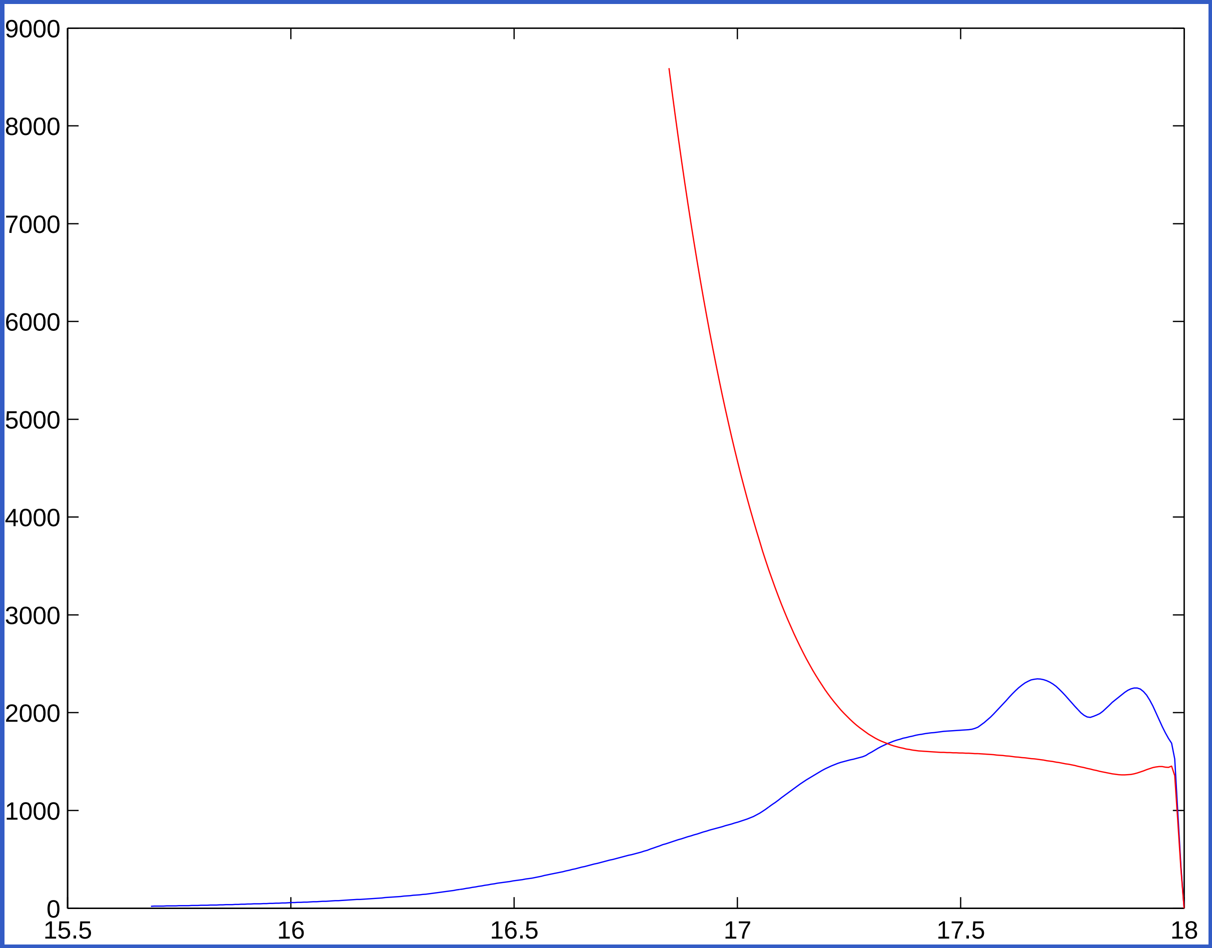
Dual Turb L2



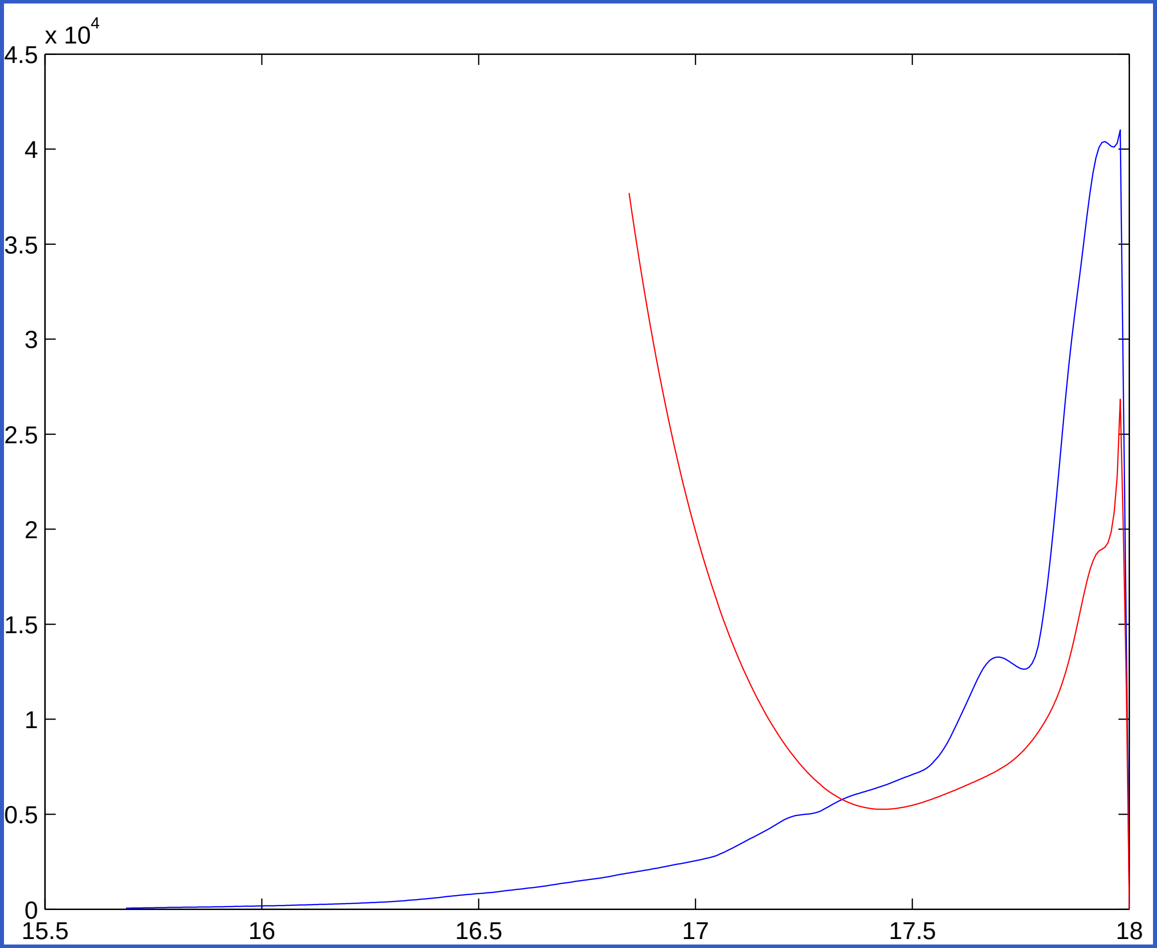
Dual Turb Lmax



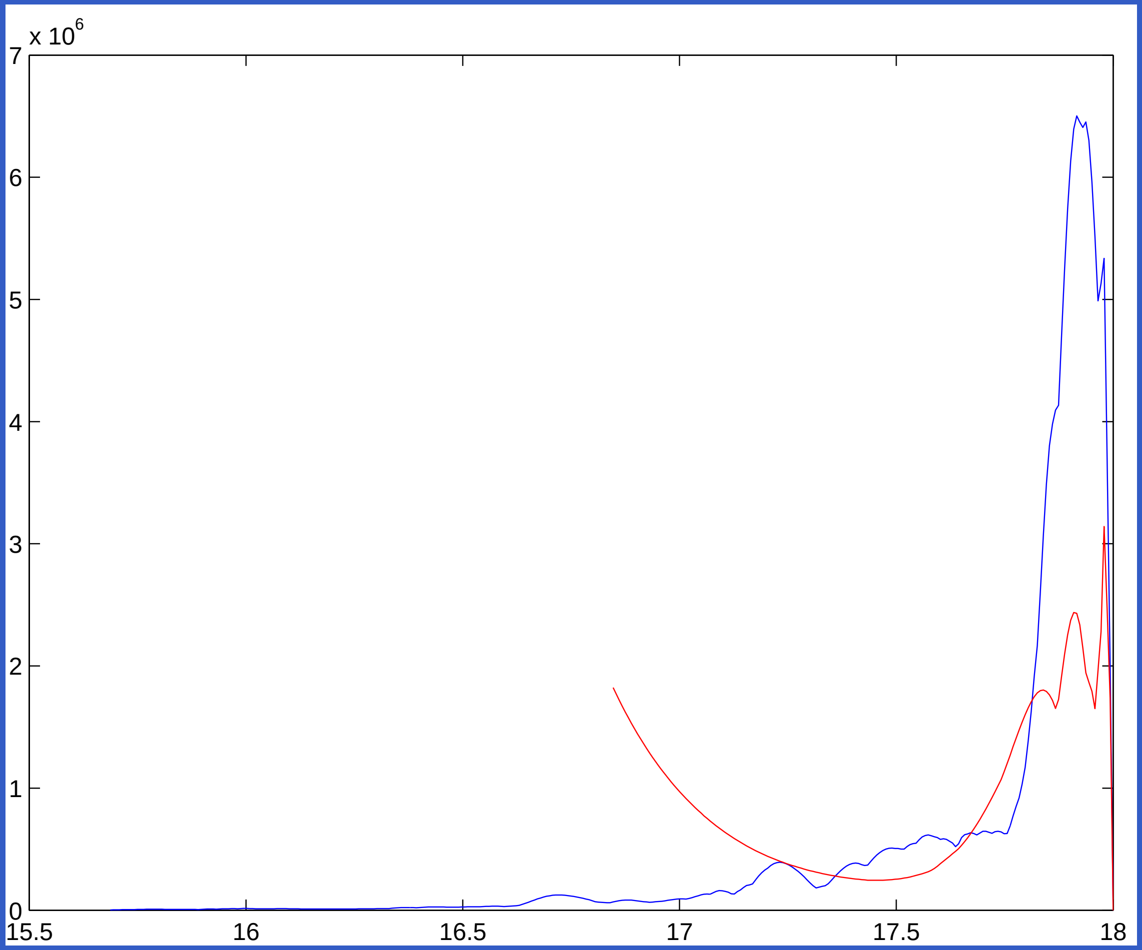
Dual Turb W11



Dual Turb W12



Dual Turb W1max



KEY FACTS

- SHOCKS/TURBULENCE: IRREVERSIBILITY
- NON-EXISTENCE of EXACT (strong) SOLUTIONS
- EXISTENCE of APPROX WEAK SOLUTION
- OUTPUT WEAK UNIQUENESS
- CANCELLATION in DUAL PROBLEM
- MIXING STABLE, UNMIXING UNSTABLE

MIXING: Statistic approach

- Unmixed $\rightarrow$ Mixed: Likely/Probable
- Ordered $\rightarrow$ Unordered: Likely/Probable
- Mixed $\rightarrow$ Unmixed: Unlikely/Unprobable
- Unordered $\rightarrow$ Ordered: UnLikely/UnProbable
- Mixing irreversible

- Main aspect: statistics

MIXING: Deterministic, Stability

- Unmixed $\rightarrow$ Mixed: Stable (under perturb)
- Mixed $\rightarrow$ Unmixed: Unstable (under peturb)
- Laminar $\rightarrow$ Turbulent: Stable (under perturb)
- Turbulent $\rightarrow$ Laminar: Unstable (under peturb)
- Mixing laminar/turb irreversible

- Smash into pieces: Easy (low precision)
- Assemble from pieces: Difficult (high precision)

STABILITY + PRECISION

- Stability vs Uniqueness vs Well-posedness
- Perturbations vs Precision
- OUTPUT UNIQUENESS vs STABILITY
- G2

IMPERFECTION of MATH?

- Non-existence of pointwise solutions
- Compare $x^2 = 2$: nonexistence of rational root x
- Collapse of Pythagorean school
- Resolution: irrational root $\sqrt{2}$: approximate solution
- Clay Prize: Existence?
- Non-Existence
- School of Analytical Mathematics based on existence of solutions
- School of Computational Mathematics: existence of approx solutions.

IMPERFECTION of NATURE?

- Newton’s Second Law not followed pointwise
- Weak-Strong G2 satisfaction?
- World = Euler/G2??
- Euler/G2 universal macro-model without parameters
- Effective viscosity, heat conductivity from Euler/G2 = ?

SUMMARY

- New Foundation of Thermodynamics
- World = Euler/G2 (parameter h), Deterministic
- Non-existence of (stable) strong solutions
- Existence of (stable) weak approximate solutions
- Least squares stab vs Irreversibility
- 2nd Law follows from 1st Law/G2.
- Output uniqueness/independence of h
- New aspect: STABILITY / OUTPUT UNIQUENESS
- Particle systems, Quantuum Mechanics ....

BODY and SOUL: www.bodysoulmath.

- Body $=$ Mathematical Computation
- Soul $=$ Matematical Analysis
- Vol I-III: Calculus Linear Algebra 2003
- Vol IV: Turbulent Incompressible Flow 2006
- Vol V : Thermodynamics 2007
- Vol VI: Solid Mechanics 2007
- ...

FEniCS: www.fenics.org

- Automation of Computational Calculus
- Solve Any PDE on Lap Top
- Automated as
- Solving $\ddot{u} + u = 0$ on Pocket Calculator
- Computing $\sin(x)$ and $\cos(x)$

FINITE ELEMENT CENTER

- Virtual Laboratory/University
- Virtual Faculty $+$ Students
- Virtual Educatioon $+$ Research
- Apply for Virtual Membership!!
- www.femcenter.org