Automatic *hp*-Adaptivity for Elliptic and Maxwell Problems

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The lecture will focus on explaining details of two versions of the fully automatic *hp*-algorithm. The original, energy-driven version aims at minimizing the discretization error measured in a global (energy) norm. The goal-driven algorithm builds upon the logic of the original version but it delivers meshes that minimize the error measured in a specific *quantity of interest* q.o.i. represented by a linear (and continuous) functional.

Both algorithms are based on a two-grid paradigm: a *coarse* mesh, and a *fine* grid obtained from the coarse one through a global (isotropic) *hp*-refinement: each element is divided into eight (3D) element sons and the order is raised uniformly by one. Given a fine grid solution $u_{h/2,p+1}$, we determine the next, optimally refined coarse grid hp_{opt} by solving the following optimization problem,

$$\frac{\|u_{h/2,p+1} - \Pi_{hp} u_{h/2,p+1}\| - \|u_{h/2,p+1} - \Pi_{hp_{opt}} u_{h/2,p+1}\|}{n_{hp_{opt}} - n_{hp}} \rightarrow \max$$

Here $\Pi_{hp} u_{h/2,p+1}$ represents the *projection-based interpolant* of fine grid solution on the current coarse grid and, similarly, $\Pi_{hp_{opt}} u_{h/2,p+1}$ is the interpolant of the same solution an on the optimal grid to be determined. The decrease in the interpolation error is measured against the increase in the problem size, n_{hp} denotes the number of d.o.f. in an *hp* grid. Thus, the one step algorithm aims at maximizing the rate with which the interpolation error decreases with refinements. Contrary to the approximation error, the interpolation error depends upon the element size h and order of approximation p *locally* which enables the construction of an effective, fully discrete optimization procedure. The search for the optimal refinements reflects the logic of the projection-based interpolation. First, optimal refinements of the coarse grid edges are determined. This includes making the choice between the p-refinement and a number of *competitive* h-refinements, as well as determining the optimal order of approximation for the edge-sons of h-refined edges. The optimal edge refinements set up the stage for finding the optimal face refinements, they essentially limit the number of considered, competing face refinements. Finding the optimal refinements for faces involves again making the choice between possible isotropic and *anisotropic* refinements and determining the optimal order of approximation for the newly created faces. The final step of the algorithm involves determining optimal refinements of the coarse grid element interiors using the same logic.

Once the optimal refinements are executed, the new coarse grid is again refined globally to yield the next fine grid solution and the procedure is repeated until an error criterion is met. *It is the fine grid solution that is delivered as the final product !* The solution of the coarse grid problem is needed only for an error estimation.

The goal-oriented version of the algorithm is based on the same logic, except that it aims at minimizing a different functional representing an upper error bound on the error in q.o.i. and involving the solutions of the original and dual fine grid problems.

The *hp*-algorithms will be illustrated with a large number of 2D and 3D examples including acoustics and Maxwell scattering problems, simulations of electromagnetic and acoustic logging devices, structures with thin-walled components and the (non-linear) full-potential equation. In particular, we shall demonstrate how the automatic *hp*-adaptivity can efficiently resolve boundary layers created by various versions of the (unsplit) $PerfectlyMatchedLayer$ in wave simulations.

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