Spectral decomposition and asymptotic stability for a class of aeroacoustic models

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A class of aeroacoustic models representing the vibrations of a fluid-structure coupled system is considered in an abstract framework. This abstract framework is illustrated by applications to various aeroacoustic models (like the Helmholtz model or more elaborated ones). Due to the coupling terms, this kind of models cannot be written in a standard way in terms of a second order evolution equation involving a self-adjoint operator. For this reason, the existence of an eigenbasis allowing a spectral decomposition of solutions is not obvious. Using a symmetrization procedure, we prove the existence of a suitable basis used in a spectral decomposition of solutions.
As an application, we consider the problem of the asymptotic behavior of the solutions of such problems when they are subject to a weak nonlinear damping. Using this suitable basis, we give a necessary and sufficient condition insuring strong asymptotic stability of the solutions under standard assumptions on the feedback. As a consequence, in the applications, the problem of asymptotic stability reduces to the proof of a uniqueness principle. The results are also stated in an abstract framework and may be applied to various aeroacoustic models (like the ones already mentioned above) with distributed, boundary or even pointwise feedbacks.
Our work is organized as follows. Firstly, we introduce an abstract aeroacoustic model for which we state a result of wellposedness. Next, we give some concrete examples of aeroacoustic models that enter in this abstract framework. One of them is the Helmholtz model which represents a solid in a cavity filled by a fluid, and the motion of the solid is represented by a harmonic oscillator. The second one is a more elaborate fluid-solid structure model describing acoustic wave propagation in a cavity filled by a fluid flow, where the structure is a membrane and its motion is represented by a wave equation. Then, we prove the existence of a suitable basis allowing a spectral decomposition of the solution of the abstract aeroacoustic model. Finally, we turn to the problem of asymptotic stability and we give a necessary and sufficient condition of asymptotic stability for the solutions of the abstract aeroacoustic model.

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