Coupling of Boundary Elements and Discontinuous Galerkin Methods

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Discontinuous Galerkin Method is a general name given to a class of numerical schemes for problems of very different type with the common feature of using spaces of piecewise polynomial functions over a triangulation, without continuity restrictions on the interfaces. When applied to elliptic problems, these are obviously non–conforming methods. A very complete taxonomy and the basic ideas for the analysis of most elements in this class of methods is given in [1].
It seems interesting to explore the possible application of non–local boundary conditions given by integral operators to discontinuous Galerkin schemes, since these boundary conditions are used to cope with unbounded domains or also with very large parts of the domain where coefficients are constant and no source terms appear. In this spirit, the paper [3] gave a first step by showing how to couple a Linearly Discontinuous Galerkin method with a classical Galerkin Boundary Element Method. The main problem of the coupling stems from the fact that the trace given by the DG scheme is not continuous and a non–conformity arises in the BEM discretization. The solution proposed in [3] consisted on copying one of the unknowns (the trace in the interface between the DG domain and the exterior) and using two different approximations of it. In this way, the DG and BEM grids act almost separately.
Using the results of [2] we will show here how to couple a BEM with the LDG approximation of an elliptic nonlinear problem. We will endeavour to generalize the basic ideas for general DG methods and boundary element discretizations and to make the most of the very uncoupled aspect of the coupled method to simplify implementation.

References

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