Existence, unicity and asymptotic analysis for solution of the Burgers equation with relaxation

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The wave propagation processes in non-linear media with relaxation are described by the following integro-differential equation [1]:

$$\frac{\partial u}{\partial x} - A\frac{\partial^2 u}{\partial y^2} = \alpha\{c\frac{\partial f(u)}{\partial y} - \frac{\varepsilon}{\tau}u + \frac{\varepsilon}{\tau^2}\int_{-\infty}^{y} u(x,y')\exp\frac{y'-y}{\tau}dy'\}, \tag{1}$$

set in the domain

$$Q_T = \{(x,y) : 0 \leq x \leq T, -\infty < y < \infty\}$$

with the initial condition

$$u(x=0,y) = \varphi(y) \tag{2}$$

and the periodicity condition

$$u \text{ — 1-periodic in } y. \tag{3}$$

Here A, c, ε, τ are constants, $\varepsilon > 0$, $\tau > 0$, f, φ are functions, $f \in C^1(\mathbb{R})$, $\varphi \in W_2^{1,per}$, where $W_2^{1,per}$ is a completion of the set $C^\infty_{1-per.}(\mathbb{R})$ of 1-periodic infinitely differentiable functions with respect to the norm $\|\cdot\|_2^1$,

$$\|\varphi(y)\|_2^1 = \left(\int_0^1 \varphi^2 + (\frac{\partial \varphi}{\partial y})^2 dy\right)^{\frac{1}{2}}.$$

By definition, set

$$\|u(x,y)\|_X^2 = \sup_{x\in[0,T]}\int_0^1 \left\{u^2(x,y) + \left(\frac{\partial u}{\partial y}\right)^2\right\} dy.$$

Let X be the completion of $C^\infty_{1-per.(y)}(\mathbb{R})$ with respect to this norm.

THEOREM *Let $f \in C^1(\mathbb{R})$, $\varphi \in W_2^{1,per}$. If $f'(u)$ satisfies a Lipschitz condition with constant L*

$$|f'(u_1) - f'(u_2)| \leq L|u_1 - u_2|$$

and if $\exists \sup_{u\in X} f'(u)$, there exists $\alpha > 0$ such that problem (1), (2), (3) *has a unique solution in X such that $< u >= 0$.*

A priori estimate for the solution is proved.

The asymptotic expansion in the case $\varepsilon \to 0$ is constructed and justified. As a consequence for any $k \in \{1,2,3...\}$ there can be calculated an approximation convergent to the solution with the order ε^k.

References

[1] O .V. Rudenko, S. I. Soluyan, *Theoretical Foundations of Nonlinear Acoustics*, Moscow: Nauka, 1975.

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