

Existence, unicity and asymptotic analysis for solution of the Burgers equation with relaxation

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The wave propagation processes in non-linear media with relaxation are described by the following integro-differential equation [1]:

$$\frac{\partial u}{\partial x} - A \frac{\partial^2 u}{\partial y^2} = \alpha \left\{ c \frac{\partial f(u)}{\partial y} - \frac{\varepsilon}{\tau} u + \frac{\varepsilon}{\tau^2} \int_{-\infty}^y u(x, y') \exp \frac{y' - y}{\tau} dy' \right\}, \quad (1)$$

set in the domain

$$Q_T = \{(x, y) : 0 \leq x \leq T, -\infty < y < \infty\}$$

with the initial condition

$$u(x = 0, y) = \varphi(y) \quad (2)$$

and the periodicity condition

$$u \text{ — 1-periodic in } y. \quad (3)$$

Here A, c, ε, τ are constants, $\varepsilon > 0, \tau > 0, f, \varphi$ are functions, $f \in C^1(\mathbb{R}), \varphi \in W_2^{1,per}$, where $W_2^{1,per}$ is a completion of the set $C_{1-per.}^\infty(\mathbb{R})$ of 1-periodic infinitely differentiable functions with respect to the norm $\|\cdot\|_2^1$,

$$\|\varphi(y)\|_2^1 = \left(\int_0^1 \varphi^2 + \left(\frac{\partial \varphi}{\partial y} \right)^2 dy \right)^{\frac{1}{2}}.$$

By definition, set

$$\|u(x, y)\|_X^2 = \sup_{x \in [0, T]} \int_0^1 \left\{ u^2(x, y) + \left(\frac{\partial u}{\partial y} \right)^2 \right\} dy.$$

Let X be the completion of $C_{1-per.(y)}^\infty(\mathbb{R})$ with respect to this norm.

THEOREM *Let $f \in C^1(\mathbb{R}), \varphi \in W_2^{1,per}$. If $f'(u)$ satisfies a Lipschitz condition with constant L*

$$|f'(u_1) - f'(u_2)| \leq L|u_1 - u_2|$$

and if $\exists \sup_{u \in X} f'(u),$ there exists $\alpha > 0$ such that problem (1), (2), (3) has a unique solution in X such that $\langle u \rangle = 0$.

A priori estimate for the solution is proved.

The asymptotic expansion in the case $\varepsilon \rightarrow 0$ is constructed and justified. As a consequence for any $k \in \{1, 2, 3, \dots\}$ there can be calculated an approximation convergent to the solution with the order ε^k .

References

- [1] O .V. RUDENKO, S. I. SOLUYAN, *Theoretical Foundations of Nonlinear Acoustics*, Moscow: Nauka, 1975.

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