Resolution of Three Scientific Mysteries by Computation

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The talk is based on material from the new book [2] to appear in the Fall 2006. A preliminary version of the book can be downloaded from www.fenics.org [8] or www.bodysoulmath.org [7].

We present resolutions of the following three mysteries, which have haunted scientists over centuries:

1. d'Alembert's Mystery: Zero drag of inviscid flow.
2. Loschmidt's Mystery: Time reversibility in Hamiltonian systems.
3. Sommerfeld's Mystery: Stability of Couette flow.

All these mysteries reflect *paradoxes*, where phenomena predicted by mathematics are not at all observed in reality. Since science is supposed to be rational and based on mathematics, paradoxes are catastrophical for the credibility of science, and thus have to be resolved (or covered up), in one way or the other, at any price [4].

1. d'Alembert's Mystery

In d'Alembert's Mystery formulated in 1752 [1], mathematics predicts that a body may move through a fluid with zero (very small) viscosity, like air and water, with zero (very small) resistance or *drag*. But everbody knows that this is impossible; the drag increases roughly quadratically with the velocity and becomes very substantial for higher velocities.
The cover up of d'Alembert's Mystery is to blame the assumption of zero viscosity for the erronous prediction: In reality there is always some possibly very very small viscosity (of some nature), which changes everything (in some mysterious way).

2. Loschmidt's Mystery

In Loschmidt's Mystery formulated in 1876 [3], mathematics of systems with zero viscosity predicts that time reversal and a perpetum mobile is possible. But everybody knows that time is always moving forward and that a perpetum mobile is impossible.
The cover up of Loschmidt's Mystery is to introduce statistical mechanics based on microscopic games of roulette.

3. Sommerfeld's Mystery

In Sommerfeld's Mystery from 1908 [6], mathematics predicts that the simplest of all flows, Couette flow with a stationary linear velocity profile, is stable and thus should exist. But nobody has ever observed this flow in a fluid with small viscosity.
The cover up of Sommerfeld's Mystery is to say that a linear velocity profile is too simple for the mathematical theory to apply [5].

4. Mystery Resolution

We will resolve all three mysteries by showing computationally that stable turbulent approximate solutions to the the inviscid incompressible Euler equations do exist, while stable exact laminar solutions do not exist.
d'Alembert's Mystery will thus be resolved by observing that d'Alembert's zero drag exact potential solution is unstable and instead a turbulent approximate solution develops with substantial drag.
Loschmidt's Mystery will be resolved by observing that the existing turbulent approximate solutions to the inviscid Euler equations are not reversible, because of substantial turbulent dissipation on all computational meshes, while exact solutions to the Euler equations would have been reversible had they only existed, but they don't.
Sommerfeld's Mystery will be resolved by computationally observing that Couette flow is unstable and by understanding that a mathematical stability analysis based on the eigenvalues of a non-normal matrix may be incorrect.

References

[1] J. d'Alembert, *Essai d'une nouvelle théorie de la résistance des fluides*, Paris 1752, http://gallica.bnf.fr/anthologie/notices/00927.htm.

[2] Johan Hoffman and Claes Johnson, *Computational Turbulent Incompressible Flow*, Vol 4 of Applied Mathematics: Body and Soul, Springer, 2006.

[3] J. Loschmidt, Sitzungsber. Kais. Akad. Wiss. Wien, Math, Naturwiss. Classe 73, 1876, pp 128-142.

[4] ...the whole procedure was an act of despair because a theoretical interpretation had to be found at any price, no matter how high that might be... until after some weeks of the most strenuous work of my life, light came into the darkness, and a new undreamed-of perspective opened up before me. (Planck on the statistical mechanics basis of his radiation law from his Nobel Lecture in 1918)

[5] H. Schlichting, *Boundary Layer Theory*, McGraw-Hill, 1955.

[6] A. Sommerfeld, *Ein Beitrag zur Erklärung der turbulenten Flüssigkeitsbewegungen*, Atti del 4. Congr. Internat.dei Mat. III, Roma, 1908, pp 116-124.

[7] *The Body&Soul Project*, bodysoulmath.org.

[8] *The FEniCS Project*, www.fenics.org.

[9] *Finite Element Center*, www.femcenter.org.

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