A robustly convergent finite element method for wave propagation in media with thin slots

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A straightforward numerical solution of a wave propagation problem in a medium containing thin slots of width $\varepsilon > 0$ requires a mesh size $h \simeq \varepsilon \ll \lambda$ to resolve the small structures – here, λ denotes the wave length. In view of the limit case $\varepsilon \to 0$, we seek to incorporate the effects of the thin slots, while maintaining a global mesh size proportional to λ, i. e. we want to avoid the small scale resolution.
We consider time-harmonic acoustic wave propagation in two space dimensions. As a first step, the unknown solution u^ε is approximated by a function $\tilde{u}^\varepsilon$, which solves a *one-dimensional* problem inside the thin slots and the original *two-dimensional* problem outside. The 1D approximation inside the thin slot is motivated by considering the behavior of the solution in a wave guide of height ε. An additional condition given on the slot interfaces matches the 1D and 2D parts of the solution.
As a second step, the problem for $\tilde{u}^\varepsilon$ is discretized with standard 1D and 2D finite elements inside the slots and outside, respectively, where the mesh size can now be chosen independently of ε. The matching condition ensures that the finite element solution $\tilde{u}^\varepsilon_h$ is continuous across the slot interfaces.
The triangle inequality is used to split the total error into an approximation error and a discretization error:

$$\|u^\varepsilon - \tilde{u}^\varepsilon_h\| \leq \|u^\varepsilon - \tilde{u}^\varepsilon\| + \|\tilde{u}^\varepsilon - \tilde{u}^\varepsilon_h\|. \tag{1}$$

The *approximation error* can be bounded by some power of ε. It has been analyzed in the first part of [1]. We shall analyze the *discretization error* here. In particular, we demonstrate that it is uniformly bounded for ε small enough. This can be done by adapting a theorem due to Mikhlin ([2], Thm. 2).
We finally conclude that the convergence of the method is *robust,* i. e. does not deteriorate as $\varepsilon \to 0$. This makes the proposed method useful for arbitrarily thin slots. The robustness shall also be illustrated via numerical examples.

References

[1] S. Tordeux, *Méthodes Asymptotiques pour la Propagation des Ondes dans les Milieux comportant des Fentes*, Thèse de Doctorat, Université de Versailles Saint-Quentin-en-Yvelines, 2004.

[2] L. Demkowicz, *Asymptotic Convergence in Finite and Boundary Element Methods: Part 1: Theoretical Results*, Comput. Math. Appl. **27** (12), pp. 69–84, 1994.

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