Conservative and well-balanced discretizations for shallow water flows on rotational domains

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We are interested in the problem of numerical simulations for shallow water flows on rotational domains. This problem is related either to some industrial applications or to climate, meteorological or oceanographic simulations. The existence of non-standard steady states and complex stability properties make this problem a great numerical challenge.

We introduce two new finite volume discretizations that are adapted to this problem.
The first one is devoted to ensure some discrete conservativity properties in order to perform accurate long time integration. The main goal is to ensure the conservativity of the discrete inertial linear momentum. The preservation of discrete stability properties for the water height (conservativity, positivity) is quite easy to obtain when using a finite volume method. It is no more the case when considering inertial momentum since in the classical formulation of the problem [1] we do not access this quantity directly. Here we propose a new method that allows us to ensure this property. The key point is to write a scheme for the relative quantities while controling the conservation of inertial quantities. In a second step we show that this method naturally leads to the formulation of a new discrete Coriolis term which is no more cell-centered but defined by using the mass fluxes at the cell interfaces. Some numerical results highlight the fact that this new discretization is more accurate than the classical one, especially when non-isotropic meshes are considered.
The second finite volume method we introduce is a *well-balanced* method for the so-called geostrophic equilibrium. Following the work of Botta et al. [2] the idea is to compute the gradient of the deviation between the pressure and a geostrophic pressure instead of the classical pressure gradient. The geostrophic pressure is computed as the solution of an auxiliary Poisson problem for which the source term comes from the Coriolis effect. Here also the correct evaluation of the Coriolis effect appears to involve the interface mass fluxes.

References

[1] Coriolis, G.G., Mémoire sur les équations du mouvement relatif des systèmes de corps. *Journal de l'école Polytechnique*, Vol 15 (1835), pp. 142–154.

[2] N. Botta, R. Klein, S. Langenberg, S. Lützenkirchen, Well-balanced finite volume methods for nearly hydrostatic flows, Volume 196, Issue 2, 20 May 2004, Pages 539-565.

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