Simulation de la propagation laser dans un plasma

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Travail avec

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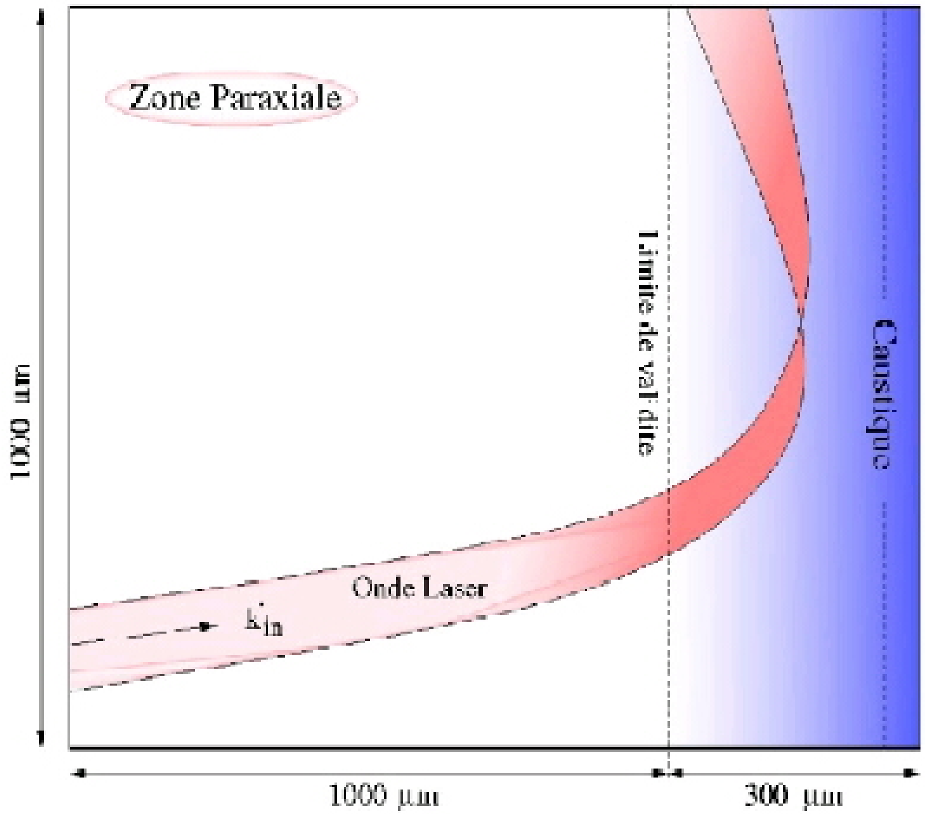
Motivation.

Propagation de faisceaux laser dans les plasmas pour le projet Laser MégaJoule

(fusion par Confinement Inertiel).

Présentation du domaine



Plan

Description des 2 modèles.

Simulation modèle paraxial.

Simulation modèle eq. ondes fréquentielle.

Conclusion.

1 Description des modèles

Modèle théorique Euler-Maxwell.

...

A. Modèle eq. ondes fréquentielles.

$N(t, \mathbf{x})$ densité électronique adimensionnée par la densité critique.

$.\mathbf{U}(t, \mathbf{x})$ vitesse du plasma.

$.\boldsymbol{\Psi}(t, \mathbf{x})$ champ laser

c, c_s, vitesse de la lumière, du son,

$2\pi/k_0$, longueur d'onde du laser.

$$
\begin{aligned}
\frac{\partial}{\partial t} N + \nabla (N\mathbf{U}) &= 0 \\
\frac{\partial}{\partial t}(N\mathbf{U}) + \nabla (N\mathbf{U}\mathbf{U}) + c_s^2 \nabla N &= -\gamma_p N \nabla |\Psi|^2
\end{aligned}
$$

$$
2i \frac{1}{c}\frac{\partial}{\partial t}\Psi + \frac{1}{k_0}\Delta \Psi + k_0(1-N)\Psi + \frac{2i}{c}\nu \Psi = 0
$$

ν, coefficient d'absorption,

γ_p coefficient dependant du type d'ion,

sachant que $\alpha^{in}(t, \mathbf{x})$ est non oscillante en $\mathbf{x}$, la condition sur le bord éclairé

$$
(\frac{\partial}{\partial \mathbf{n}} - ik_0|\mathbf{K}^{in}|) \left(\Psi - e^{ik_0 \mathbf{K}^{in}.\mathbf{x}} \alpha^{in}\right) = 0, \qquad \mathbf{K}^{in} = \mathbf{e}_b \sqrt{1 - N^{in}}, \qquad \mathbf{e}_b \text{fixed}
$$

On pose $\mathbf{x} = (x, y)$.

Hyp. pour simulations $\quad N(t, x, y) = N_0(t, x) + \delta n(t, x, y), \quad \delta n \ll N_0$

.B. Modèle Paraxial

Hyp. sup. : N_0 fonction très faiblement variable en espace, indépendante du temps,

Approximation W.K.B., (equation eikonale): enveloppe spatiale

$$\mathbf{\Psi}(t, \mathbf{x}) = e^{ik_0 \mathbf{K}.\mathbf{x}} E(t, \mathbf{x}), \qquad |\mathbf{K}(\mathbf{x})| = \sqrt{1 - N_0}$$

$$i \left(\frac{2}{c} \frac{\partial}{\partial t} + 2\mathbf{K}.\nabla + \nabla.\mathbf{K} + \frac{2\nu}{c} \right) E + \frac{1}{k_0} \mathbf{\Delta}_{\perp}^{K} E - k_0 (N - N_0) E = 0$$

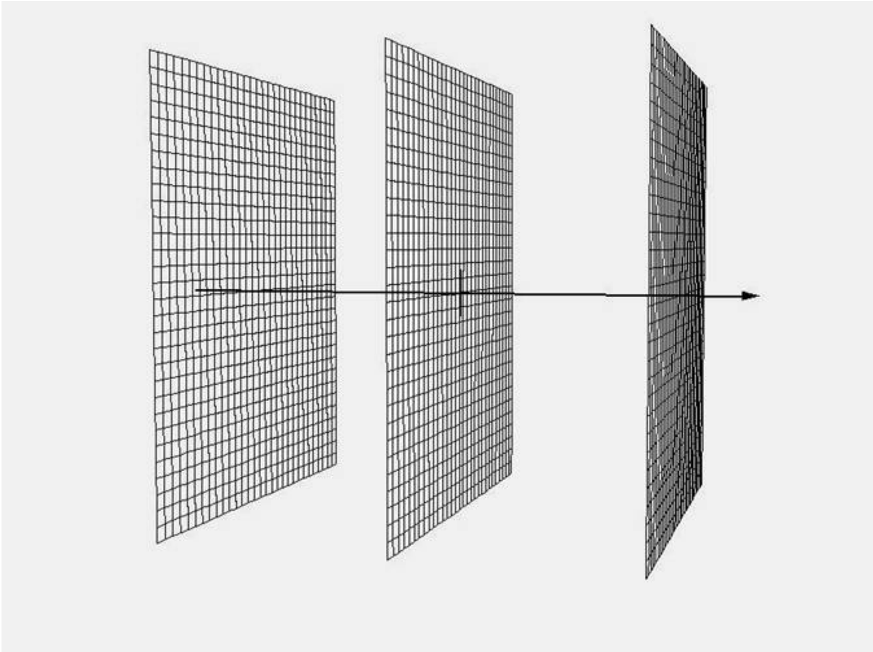
$$\text{où : } \mathbf{\Delta}_{\perp}^{K} \bullet = \nabla.[(\overline{\overline{1}} - \frac{\overline{\overline{\mathbf{K}\mathbf{K}}}}{|\mathbf{K}|^2}) \nabla \bullet]$$

avec une bonne condition sur le bord entrant.

2 Simulation avec le modèle paraxial

Pour Euler, méthode Lagrange + advection.

Pour Paraxial, méthode marche en espace avec FFT.



3 Simulation modèle eq. ondes fréquentielle

Implicit time discretization $\Rightarrow$ Helmholtz problem (finite difference type).

Boundary conditions PML, for $y = 0$ and $y = y_{max}$.

Difficulties

- Multiscale problem. Spatial step for Helmholtz 5 or 10 times smaller than the spatial step for Euler
- Coupling the Euler equations with the Helmholtz $\Rightarrow$ interpolation between the two grids
- Helmholtz problem with variable coefficients in a non symmetric form (due to PML)
- Realistic computation $\Rightarrow$ hundreds of millions of unknowns

Principle. Thanks to $\delta N(x, y) << N_0(x)$, solution of

$$k_0^{-2}\Delta\psi + \mathrm{i}\mu\psi + (1 - N_0(x))\psi - \delta_N(x, y)\psi = \mathrm{i}\mu\psi^{pre}$$

by a iterative Krilov method, with a preconditioning by

$$k_0^{-2}\Delta\psi + \mathrm{i}\mu\psi + (1 - N_0(x))\psi = \ldots$$

The linear system to be solved has the following form : beside the interior domain, two small zones for to the PML.

$$\begin{pmatrix} P_1 & C_1 & 0 \\ C_3 & B + D_{\delta N} & C_4 \\ 0 & C_2 & P_2 \end{pmatrix} \psi = F, \tag{1}$$

$$\text{with : } A_D = \begin{bmatrix} P_1 & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & P_2 \end{bmatrix}, \text{ and } A_E = \begin{bmatrix} 0 & C_1 & 0 \\ C_3 & D_{\delta N} & C_4 \\ 0 & C_2 & 0 \end{bmatrix}.$$

- Remarks
  - The matrices P_1 , P_2 are factorized by a direct method
  - B is a very large matrix but with a simple structure

Cyclic Reduction for solving $Bu = f$ (1/2)

$$Bu = \begin{bmatrix} A & -T & & & \\ -T & A & -T & & \\ & \ddots & \ddots & \ddots & \\ & & & -T & A \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix}$$

where $T = cI$ and u_k vectors of dimension n_y (simulation domain n_xxn_y points).

Recursively and in parallel:

To be solved :
$$\begin{array}{ccccccccccc} -Tu_{i-2} & + & Au_{i-1} & - & Tu_i & & & & & = & f_{i-1} \\ & - & Tu_{i-1} & + & Au_i & - & Tu_{i+1} & & & = & f_i \\ & & & - & Tu_i & + & Au_{i+1} & - & Tu_{i+2} & = & f_{i+1} \end{array}$$

Reduction : $-TA^{-1}Tu_{i-2} + (A - 2TA^{-1}T)u_i - TA^{-1}Tu_{i+2} = \ldots$

Redistribution : $u_{i-2}, u_i \Longrightarrow u_{i-1}$

Cyclic Reduction for solving $Bu = f$ (2/2)

- Via a LR (Parlett) diagonalization process (much cheaper than classical QR method):

$$A = Q\Lambda^{(0)}Q^T, \qquad T = Q\Gamma^{(0)}Q^T \text{et} QQ^T = I$$

- Induction formulas

$$\begin{aligned} T^{(r)} &= \left(T^{(r-1)}\right)^2 \left(A^{(r-1)}\right)^{-1} \\ A^{(r)} &= \left(A^{(r-1)}\right)^{-1} - 2T^{(r)} \end{aligned} \Longrightarrow \begin{aligned} \Gamma^{(r)} &= \left(\Gamma^{(r-1)}\right)^2 \left(\Lambda^{(r-1)}\right)^{-1} \\ \Lambda^{(r)} &= \left(\Lambda^{(r-1)}\right)^{-1} - 2\Gamma^{(r)} \end{aligned}$$

- Elimination

$$x = (A^{(r-1)})^{-1}(y + T^{(r)}z) \Longrightarrow X = (\Lambda^{(r-1)})^{-1}(Y + \Gamma^{(r)}Z)$$

- Redistribution

Constraints

- Storage of the full $nx \times nx$ complex matrix Q
- Efficient matrix-vector products

Computer implementation

- Platform **HERA** (C++)
- BLAS routines
- Complex subroutine
- Hybrid MPI (internode) / Multithreading pthread (intranode)

CPU time per GMRES iteration

Increased size problem: the number of points in both directions are doubled

Nb Procs	4	16	64	256
# d.o.f. $\times 10^6$	1.6	6.3	25.4	101.6
CPU LR	$3s$	$12s$	$48s$	$189s$
CPU per GMRES iteration	1	2.1	4.2	8.5

- CPU time $\sim n_x^2 n_y$

Numerical simulations

- $L_x = 2000\ \lambda_0$, $L_y = 2000\ \lambda_0$
- 10 points per wavelength in the Helmholtz zone
- 200 millions unknowns in the Helmholtz zone, 16 millions fluid unknowns.
- Density N_0 linear from 0.1 to 0.95 (critical density)

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ON THE LARGEST COMPUTER CENTER in France

- 200 processors.
- 240s per time iteration (about 12 Krilov iterations)
- Laser simulated during a physical time of 22 ps
- Elapsed time for the full simulation: 20 hours.

Conclusions

Quand δ_N augmente, le nombre d'iterations de Krilov augmente.

Les problèmes d'optique pour les plasmas sont un vrai challenge de calcul scientifique.

Problèmes liés aux instabilités Raman et Brillouin.